# The Intertemporal Keynesian Cross

Adrien Auclert[*] Matthew Rognlie[†] Ludwig Straub[‡]

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**Abstract**

We generalize the traditional, static Keynesian cross by deriving an *intertemporal Keynesian cross* for the dynamic output response to government spending and taxes in microfounded general equilibrium models. Intertemporal marginal propensities to consume (iMPCs) are sufficient statistics for this response, with fiscal multipliers depending only on the interaction between iMPCs and public deficits. We provide empirical estimates of iMPCs and argue that they are inconsistent with representative-agent or two-agent models, but can be matched by certain heterogeneous-agent models. Models that match empirical iMPCs imply larger and more persistent output responses to deficit-financed fiscal policy, with cumulative spending multipliers above one.

[*]Stanford University, CEPR and NBER. Email: aauclert@stanford.edu.

[†]Northwestern University and NBER. Email: matthew.rognlie@northwestern.edu.

[‡]Harvard University, CEPR and NBER. Email: ludwigstraub@fas.harvard.edu.

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# 1 Introduction

How do changes in government spending and taxes affect aggregate economic activity? One of the oldest answers to this question is given by the Keynesian cross, a staple of undergraduate macro textbooks. The Keynesian cross is derived by assuming that aggregate consumption is a function only of current after-tax income, resulting in a simple equation for aggregate output:

$$dY_t = dG_t - mpc \cdot dT_t + mpc \cdot dY_t \tag{1}$$

Here, a fiscal shock to government spending $dG_t$ or taxes $dT_t$ only affects contemporaneous output $dY_t$, and transmission is determined entirely by the aggregate marginal propensity to consume out of income, $mpc$. The direct impulse from fiscal policy to aggregate demand is $dG_t - mpc \cdot dT_t$, summing the effect of higher government spending and that of lower consumption from higher taxes. It is amplified in equilibrium by the multiplier $1/(1-mpc)$, which reflects the feedback $mpc \cdot dY_t$ from output, and thus income, back to consumption.

From a modern perspective, the static consumption function underlying the Keynesian cross has serious flaws. It does not respect intertemporal budget constraints and therefore cannot be microfounded in a dynamic model. It ignores the downward pressure on consumption from anticipation of future taxes if spending is deficit-financed; inversely, it also ignores any positive effect of past income on spending today via accumulated savings. This matters because, following the pioneering work of Modigliani and Brumberg (1954) and Friedman (1957), a large body of empirical work has shown that past and expected future income affect current consumption. In light of these arguments, the literature has moved to dynamic models where consumption is the outcome of optimizing behavior.

We show that, in these modern models, there instead exists an *intertemporal Keynesian cross*. Assuming that monetary policy stabilizes the real interest rate, aggregate consumption is given by an intertemporal consumption function $\mathcal{C}_t(\{Y_s - T_s\})$, which now depends on past and future, in addition to current, income. The impulse response of output $d\mathbf{Y} = \{dY_t\}$ to a change in fiscal policy $d\mathbf{G} = \{dG_t\}$, $d\mathbf{T} = \{dT_t\}$ now solves the infinite vector-valued equation:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T} + \mathbf{M} \cdot d\mathbf{Y} \tag{2}$$

where $\mathbf{M} \equiv [M_{ts}]$ is the infinite matrix of partial derivatives $M_{ts} \equiv \partial \mathcal{C}_t / \partial Y_s$ of the intertemporal consumption function.[1] For given dates $t$ and $s$, $M_{ts}$ captures the response of consumption at date $t$ to an aggregate income shock at date $s$. These *intertemporal MPCs*, or *iMPCs*, generalize the static $mpc$: in fact, the static $mpc$ is usually estimated by looking at the immediate consumption response $M_{00} = \partial \mathcal{C}_0 / \partial Y_0$ to a surprise in income. Together, the intertemporal MPCs fully characterize the transmission from fiscal shocks to output. The logic of (2) is similar to (1), with the direct impulse

[1] Throughout this paper, we use braces $\{dY_t\}$ to denote sequences $\{dY_0, dY_1, \ldots\}$; and brackets $[M_{ts}]$ to denote infinite matrices with entries $M_{ts}$ where the first subscript indexes rows, $t = 0, 1, \ldots$ and the second columns, $s = 0, 1, \ldots$. We use boldface $d\mathbf{Y}$ and $\mathbf{M}$ when for the resulting infinite vectors and matrices.

from fiscal policy to demand being $d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T}$, which is then amplified by feedback $\mathbf{M} \cdot d\mathbf{Y}$ from output to consumption. Critically, both the impulse effect and the feedback effect at any date now depend on the full set of iMPCs and, through those, on past and expected future income and taxes.

Given that iMPCs play a central role in this theory, we look for empirical evidence to discipline them. We focus on $M_{t0} = \partial \mathcal{C}_t / \partial Y_0$—the dynamic response to an unanticipated income shock—which is where we have the best data. Our main evidence on $M_{t0}$ comes from the dynamic response to lottery earnings in Norwegian administrative data, as reported by Fagereng, Holm and Natvik (2021). These data confirm the common finding in the literature that the average static MPC $M_{00}$ is high—above 0.51 at an annual level. The key new fact we uncover is that iMPCs in subsequent years are sizable as well, with $M_{10}$ around 0.18. We show that this fact is corroborated by evidence from the 2016 Italian Survey of Household Income and Wealth (Jappelli and Pistaferri 2020).

What models can match these patterns? Representative-agent models fail immediately on the grounds that they cannot match the high static MPC $M_{00}$. This is not an issue for two-agent models with a mix of savers and hand-to-mouth spenders, which are sufficiently flexible to allow for an arbitrary static $M_{00}$. These models, however, predict a very low subsequent iMPC $M_{10}$, almost an order of magnitude below our estimate. A much better fit is achieved by heterogeneous-agent models with limited liquidity, where many households have short but nonzero effective planning horizons. This leads them to save part of an unexpected income shock for consumption in the next few years, matching the evidence on $M_{10}$.

How important is this fit to intertemporal MPCs for the effects of fiscal policy? The answer turns out to depend on the degree of deficit financing. When fiscal policy runs a balanced budget, iMPCs are in fact irrelevant: the intertemporal Keynesian cross implies a multiplier of exactly one, irrespective of iMPCs.[2] In this case, the distinction between representative-agent, two-agent, and heterogeneous-agent models does not matter.

In contrast, as we highlight in table 1, iMPCs play a pivotal role for deficit-financed fiscal shocks. We show that for any fiscal policy that involves deficits, the fiscal multiplier is determined entirely by the interaction between iMPCs and the path of primary deficits. In representative-agent models, which feature low and flat iMPCs, the fiscal multiplier on government spending remains equal to 1.[3] In two-agent models, which match the static MPC $M_{00}$ but not subsequent iMPCs, the multiplier is given by the static Keynesian cross (1), and so the impact multiplier $dY_0/dG_0$ can be significantly above 1. However, because the government must respect an intertemporal budget constraint, the cumulative multiplier—the ratio $\sum(1+r)^{-t}\,dY_t / \sum(1+r)^{-t}\,dG_t$ of the present value of the output response to the present value of the spending shock—is still equal to 1, as the eventual contraction in the future from higher taxes exactly offsets the initial boom. On the other hand, in models that match an elevated $M_{10}$, such as heterogeneous-agent models, deficit-financed spending has a persistent positive effect on output, and the cumulative multiplier

[2]This generalizes the balanced-budget multiplier result from static IS-LM models (Gelting 1941, Haavelmo 1945).

[3]See, eg, Woodford (2011) and Bilbiie (2011). This is also a consequence of Ricardian equivalence, which holds in that model.

Table 1: Government spending multipliers in the intertemporal Keynesian cross

| Fiscal rule | Multiplier | Rep. agent (RA) doesn't match MPC | Two agents (TA) matches MPC | Het. agents (HA) matches iMPCs |
|---|---|---|---|---|
| **balanced budget** | impact | 1 | 1 | 1 |
| | cumulative | 1 | 1 | 1 |
| **deficit financing** | impact | 1 | $> 1$ | $> 1$ |
| | cumulative | 1 | 1 | $> 1$ |

is strictly above 1.

The intuition for this result is that intertemporal MPCs are an additional source of feedback from output back into consumption. Deficit-financed spending leads to an increase in income without an immediately offsetting increase in taxes. Households spend this income both today and in the future, leading to an output boom in the future that triggers its own intertemporal consumption feedback, and so on. The result is a more persistent output effect, with additional amplification leading to a larger cumulative multiplier.

We consider two specific heterogeneous-agent models that can achieve a good fit to the first few iMPCs: first, a standard incomplete markets model a la Bewley (1980), where households smooth consumption by holding assets in a single liquid account, and second, a "two-account" model where households hold assets in both a liquid and illiquid account, as in Kaplan and Violante (2014) and Bayer, Luetticke, Pham-Dao and Tjaden (2019). We call these models HA-one and HA-two, respectively. In line with their higher entries off the diagonal of the $\mathbf{M}$ matrix, both models have cumulative multipliers on deficit spending that are well above one, but HA-one has significantly larger multipliers than HA-two. We trace this to a subtle difference in iMPCs between the two models. In HA-one, iMPCs continue to decline rapidly after several years, as any remaining above-normal liquidity is spent down. This feature of the model means that the cumulative MPC over several years is near one, producing very large Keynesian multipliers. In HA-two, by contrast, iMPCs decline more slowly after several years, because at that point, unspent savings have mostly migrated to the illiquid account. Although our evidence on iMPCs is not precise enough to select between these models directly, we argue that the HA-two model is more consistent with the Norwegian evidence on asset accumulation after lottery earnings.

The assumptions underlying the intertemporal Keynesian cross, although consistent with many different consumption-savings models, are special in other dimensions: a constant real interest rate, no capital, and sticky wages but flexible prices.[4] Once we relax these assumptions, iMPCs out of income are no longer sufficient statistics for the general equilibrium effect of fiscal policy: now, the consumption response to real interest rates and the iMPCs out of a surprise capital gain also matter. We prove, however, that under two assumptions on preferences and initial portfolios, there is an analytical relationship between these objects, such that iMPCs out of income and cap-

[4] We show in appendix B how these forces can be included analytically in the intertemporal Keynesian cross (2).

ital gains still fully characterize aggregate household behavior conditional on the non-household parts of the model. We show that the HA-two model is unique, among the models we consider, in its ability to fit empirical evidence on spending out of capital gains in addition to income.

With this result in hand, we conclude the paper by studying the effects of fiscal policy in a rich quantitative environment with a Taylor rule, capital investment, and sticky prices. In this environment, there are dampening effects that substantially shrink the output response to fiscal policy: inflation, exacerbated by the supply-side effects of distortionary taxation, triggers a contractionary monetary response, which pushes down both consumption and investment. The key lessons of the intertemporal Keynesian cross, however, continue to hold. A representative-agent model has low multipliers; and while a two-agent model delivers a substantial impact multiplier on deficit spending (in this case, $\sim 1.2$), its cumulative multiplier is much smaller, $\sim 0.5$. Only the HA-two model, which matches iMPCs out of income and capital gains, implies substantial multipliers both on impact and cumulatively, equal to $\sim 1.3$ for both.

Throughout the paper, we consider several analytical alternatives to heterogeneous-agent models. We show that one popular model, the bond-in-the-utility (BU) model (eg, Michaillat and Saez 2021), fails to match intertemporal MPCs, because when calibrated to match $M_{00}$, it predicts an $M_{10}$ that is too high relative to the data. We next introduce the "TABU" model, a two-agent (TA) model that mixes hand-to-mouth and BU households, and show that it can simultaneously match $M_{00}$ and $M_{10}$. We prove that its $\mathbf{M}$ matrix has the same first column as another popular analytical model, the zero-liquidity (ZL) limit of the one-account heterogeneous-agent model (e.g. Krusell, Mukoyama and Smith 2011, Werning 2015, Ravn and Sterk 2017).

Because of this result, once calibrated to $M_{00}$ and $M_{10}$, both TABU and ZL have similar iMPCs, and therefore similar fiscal multipliers, to the HA-one model with low but nonzero liquidity. Our analytical solution for TABU and ZL multipliers shows why cumulative multipliers are high under deficit financing: future debt enters directly into the formula for current output. It also shows that cumulative multipliers rise with the calibrated $M_{10}$—providing an analytical counterpart to our quantitative finding on the importance of intertemporal MPCs. At the same time, we find that these analytical models have an important limitation: they imply iMPCs out of capital gains that are far too large.

A large literature studies fiscal multipliers (see Hall 2009, Ramey 2011, and Ramey 2019 for surveys). Beyond the Keynesian cross, the first generation of microfounded models assumed a representative agent, and studied questions ranging from the role of the neoclassical wealth effect on labor supply (Aiyagari, Christiano and Eichenbaum 1992, Baxter and King 1993) to the role of monetary policy (Christiano, Eichenbaum and Rebelo 2011, Woodford 2011). By assuming constant real interest rates, we shut down these effects to focus on the feedback from income to consumption, and then reintroduce them in our quantitative model.

A second generation of microfounded models, building on Campbell and Mankiw (1989), augmented the representative-agent model with a fraction of hand-to-mouth households, obtaining "two-agent" models that could explain positive consumption multipliers in the data (eg Galí, López-Salido and Vallés 2007). As Coenen et al. (2012) documents, this modeling approach remains dominant at central banks. We show that this approach is close to the static Keynesian cross (1), echoing Bilbiie (2020). We also show that this class of models fails to match empirical iMPCs.

A recent "HANK" literature revisits stabilization policy with heterogeneous agents. Many prominent early papers in this literature focused on monetary policy (McKay, Nakamura and Steinsson 2016, Kaplan, Moll and Violante 2018, Auclert 2019). As shown by Werning (2015), models in this literature and representative-agent models can have similar predictions for the aggregate effects of monetary policy. We show, by contrast, that these models have vastly different predictions for deficit-financed fiscal policy.

Other papers have studied fiscal policy in heterogeneous-agent frameworks with nominal rigidities. Oh and Reis (2012) was an early paper studying the effect of fiscal transfers. McKay and Reis (2016) focus on the role of automatic stabilizers. Ferriere and Navarro (2024) stress the importance of the distribution of taxes for fiscal multipliers when there is heterogeneity in labor supply elasticities. Closest to our work is Hagedorn, Manovskii and Mitman (2019), who also study spending multipliers in a model with nominal rigidities similar to ours. Their analysis is based on a different equilibrium selection criterion that relies on a long-run nominal debt anchor, following Hagedorn (2016). Both our studies conclude that deficit-financed fiscal multipliers can be significantly larger than one, and that balanced budget fiscal multipliers tend to be smaller.

Since this paper was first circulated, a literature has emerged studying fiscal policy with tractable models that better match iMPCs, including Bilbiie (2024), Cantore and Freund (2021), and Angeletos, Lian and Wolf (2023). An open question is whether these models capture the same key forces as richer heterogeneous-agent models. As we discuss in the paper, the verdict is mixed. Existing models are essentially isomorphic to either TABU or ZL, which do a very good job of matching the one-account heterogeneous-agent model, but imply different long-run behavior from the two-account model and far too much spending out of capital gains. We speculate that a TABU model enhanced with additional types that are heterogeneous in asset holdings and MPCs, similar to Auclert, Rognlie and Straub (2023c), might achieve a better fit.

There is also a vast empirical literature on fiscal multipliers based on aggregate macroeconomic evidence. As surveyed by Ramey (2019), this literature points to output multipliers in the range of 0.6–0.8, though the data does not reject multipliers as high as 1.5 (Ramey 2011, ben Zeev and Pappa 2017). The literature testing state dependence has mostly focused on the prediction from the representative-agent literature that multipliers differ depending on the extent of the monetary policy response (Auerbach and Gorodnichenko 2012, Ramey and Zubairy 2018). A robust prediction of our heterogeneous-agent model is that multipliers also depend on the extent to which spending is deficit-financed. While the empirical literature acknowledges the potential importance of deficits, this prediction has not been subject to extensive testing. One intriguing new case study may be the large pandemic-associated deficits of 2020–21. These deficits have been followed by persistently high aggregate demand and inflation, consistent with high intertemporal MPCs.

Finally, our paper builds upon several lines of research that seek to discipline macroeconomic models with heterogeneity. One literature identifies sufficient statistics for partial equilibrium effects (see, for instance, Kaplan and Violante 2014, Berger et al. 2018 and Auclert 2019). This paper shows that intertemporal MPCs are sufficient statistics for both partial and general equilibrium—both in the simpler case of the intertemporal Keynesian cross, where they entirely characterize the transmission of fiscal policy, and also in a quantitative environment, where they continue to summarize aggregate household behavior.

The paper proceeds as follows. Section 2 describes the environment in which we derive the intertemporal Keynesian cross, and provides a condition for existence and uniqueness of solutions. Section 3 presents empirical evidence on iMPCs. Section 4 lays out models of the intertemporal consumption function and discusses their consistency with the iMPC evidence. Section 5 solves for fiscal multipliers using the intertemporal Keynesian cross. Section 6 shows why iMPCs remain important to discipline models of consumption in broader quantitative models. Section 7 quantifies fiscal multipliers in such a model. Section 8 concludes.

## 2 From the static to the intertemporal Keynesian cross

### 2.1 The static Keynesian cross

Starting with the *General Theory* (Keynes 1936), research in the IS-LM tradition postulates that aggregate consumption expenditure $C_t$ at any date $t$ is a certain static function $\mathcal{C}$ of aggregate after-tax income $Y_t - T_t$, the difference between aggregate output $Y_t$ and taxes $T_t$ at that same date $t$:

$$C_t = \mathcal{C}\left(Y_t - T_t\right) \tag{3}$$

In equilibrium, aggregate output $Y_t$ must equal aggregate expenditure. When the latter is made up of consumption and government spending $G_t$, this condition reads $C_t + G_t = Y_t$.

Combining these two equations, we obtain an equation for the level of aggregate income $Y_t$:

$$\mathcal{C}\left(Y_t - T_t\right) + G_t = Y_t \tag{4}$$

This equation is traditionally used to analyze the effect of a small perturbation $dG_t$, $dT_t$ to government spending or taxes on output $dY_t$, around a steady state in which $G$, $T$ and $Y$ are constant:

$$dY_t = dG_t - mpc \cdot dT_t + mpc \cdot dY_t \tag{5}$$

where $mpc = \mathcal{C}'\left(Y - T\right)$ is the economy's *marginal propensity to consume*—the derivative of the consumption function at the steady state.

Equation (5) is the well known, static "Keynesian cross": it captures the simple idea that an increase in government spending or a decline in taxes raises private income, which in turn leads to an increase in private spending, demand, and income. Assuming that $mpc < 1$, this equation can be used to solve for the output response $dY_t$ to the fiscal shock $dG_t$, $dT_t$:

$$dY_t = \frac{dG_t - mpc \cdot dT_t}{1 - mpc} \tag{6}$$

Given the shock, the magnitude of the response is entirely determined by $mpc$. Intuitively, the shock generates a first-round impulse to demand $dG_t - mpc \cdot dT_t$, which is then multiplied by $\frac{1}{1-mpc}$ due to further rounds of private spending: every additional dollar spent is an additional dollar earned, which further pushes up spending by $mpc$ dollars and translates into even higher income, and so on. Summing up the rounds, the multiplier in (6) is:

$$1 + mpc + mpc^2 + \cdots = \frac{1}{1 - mpc} \tag{7}$$

The static Keynesian cross has been extremely influential and is still routinely used by fiscal policy analysts, who pay close attention to empirical estimates of the $mpc$. Its core assumption of a static consumption function (3), however, has been severely criticized on both theoretical and empirical grounds. In response to these criticisms, modern macroeconomics has turned to microfounded models.

### 2.2 Microfounding an intertemporal consumption function

We now show how a microfounded economy gives rise to an intertemporal generalization of the static consumption function underlying the Keynesian cross.

Consider an economy inhabited by a mass 1 of agents, labeled by $i$, with an infinite horizon and perfect foresight over aggregate variables.[5] The only available asset in the economy is a real bond paying the real interest rate $r_t$ between time $t$ and time $t+1$. Production $Y_t$ is linear in effective labor $N_t$, $Y_t = N_t$, and there is perfect competition with flexible prices in the goods market. It follows that the real wage $w_t = W_t/P_t$ (per unit of effective labor) is constant and equal to 1, and that price inflation $\pi_t = (P_t - P_{t-1})/P_{t-1}$ is the same as wage inflation $\pi_t^w$ at all times $t$.

At time $t$, agent $i$ works $n_{it}$ hours. Each of these hours provides $e_{it}$ units of effective labor, so that aggregate hours are $N_t = \int e_{it} n_{it} di$. We assume that the nominal wage $W_t$ (per unit of effective labor) is partially rigid. Agents are off their labor supply curves in the short run and instead take their hours $n_{it}$ as given.[6] For now, we assume a proportional allocation rule for labor hours, with $n_{it} = N_t$.[7] We also assume a progressive retention function, as in Heathcote, Storesletten and

[5] Perfect foresight is not a limitation: since we restrict our attention to small shocks, the economy could alternatively face aggregate risk, and our results would apply to its first-order perturbation solution in aggregates (see e.g. Boppart, Krusell and Mitman 2018 and Auclert, Bardóczy, Rognlie and Straub 2021a).

[6] We assume sticky wages and flexible prices, rather than sticky prices and flexible wages. While the latter assumption is common in the representative-agent New Keynesian literature, the former has more desirable properties when combining heterogeneous agents and nominal rigidities, since it avoids countercyclical profits and large income effects on labor supply (Broer, Hansen, Krusell and Öberg 2020, Auclert, Bardóczy and Rognlie 2023a).

[7] This imposes the normalization $\int e_{it} di = 1$. A more general proportional rule where $n_{it} = n(e_{it})N_t$, with $n$ some function of $e_{it}$, is equivalent to redefining the $e_{it}$ to include $n$, so we use $n_{it} = N_t$ for simplicity.

Violante (2017), so that agent $i$'s after-tax income $z_{it}$ is given by:

$$z_{it} = \tau_t \left(w_t e_{it} n_{it}\right)^{1-\theta} = \left(Y_t - T_t\right) \cdot \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di} \tag{8}$$

where $\tau_t$ is the time-varying intercept of the retention function and $\theta \in [0,1]$ a constant progressivity parameter. The second equality in (8) follows from the definition of total taxes, $T_t \equiv w_t N_t - \int z_{it} di$, as well as $Y_t = N_t = w_t N_t$. We relax both the proportional allocation assumption and the functional form of the retention function in section 2.5.

We allow for a general class of models of consumption and saving behavior: agents can have arbitrary preferences, income processes, and constraints on asset positions. Appendix A.1 describes this class formally; section 4 offers eight specific examples. All models have in common that agents make their decisions taking interest rates $r_t$ and after-tax incomes $z_{it}$ as given and respect budget constraints. Agent $i$'s consumption $c_{it}$ and asset position $a_{it}$ (the amount of real bonds $i$ holds at the end of period $t$) satisfy:

$$c_{it} + a_{it} = \left(1 + r_{t-1}\right) a_{it-1} + z_{it} \tag{9}$$

with $a_{it}$ remaining bounded at all times.

Monetary policy follows the rule $1 + i_t = (1 + r)(1 + \pi_{t+1})$ for the nominal interest rate $i_t$, where $r$ is the steady-state real interest rate. Financial market arbitrageurs, as described in appendix A.2, enforce the Fisher equation $1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$. In equilibrium, the path for the real interest rate is therefore constant at $r_t = r$ for all $t$.[8] This constant-real interest rate rule can be viewed as a Taylor rule with a coefficient of 1 on expected inflation and provides, intuitively, a middle ground between loose policy (like at the zero lower bound) and tight policy (like with an active Taylor rule). We formalize this intuition in section 7.3.

In this environment, the only time-varying aggregate sequence that matters for agent $i$'s consumption is *aggregate post-tax income* $Z_t \equiv Y_t - T_t$. $Z_t$ pins down individual income $z_{it} = Z_t \cdot \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di}$ in (8). Hence, starting from the steady-state distribution of agents over their state variables at date 0,[9] we can express aggregate consumption at date $t$ as a function of the form:

$$C_t = \mathcal{C}_t\left(\{Z_s\}_{s=0}^{\infty}\right) \tag{10}$$

taking in the full sequence $\{Z_s\}_{s=0}^{\infty}$ of aggregate after-tax income. Similarly, there must exist an aggregate asset function $A_t = \mathcal{A}_t\left(\{Z_s\}_{s=0}^{\infty}\right)$ mapping $\{Z_s\}_{s=0}^{\infty}$ to sequences of aggregate assets

[8] Since all wealth is invested in real bonds, the ex-post real return received by households on their assets at time $t$ is always equal to $r_{t-1}$. In particular, there are no valuation effects at time 0, with households earning the steady state real interest rate $r$ on their assets at that time.

[9] While (10) can be derived assuming any fixed initial distribution, here we are interested in an MIT shock where the economy starts from a steady state with no aggregate risk, implying a steady-state initial distribution. This is consistent with interpreting our linearized model as giving impulse responses in a stochastic economy (see footnote 5). In appendix E.5, we consider a state-dependent exercise where we start the economy at a different distribution.

$\{A_t\}_{t=0}^{\infty}$. Appendix A.1 derives these results formally for our class of consumption-saving models. Different underlying primitives of these models map into different functions $\mathcal{C}_t(\cdot)$ and $\mathcal{A}_t(\cdot)$.[10]

The *intertemporal consumption function* (10) is similar to the static Keynesian consumption function (3). Both map after-tax incomes into consumption. The key difference is that the intertemporal consumption function is consistent with microfoundations and respects budget constraints. For this reason, the entire time path of income, rather than just current income, matters for determining consumption. For example, any unspent past income is allowed to be saved and to increase future consumption in (10), but not in (3). Likewise, agents can potentially borrow and spend today out of anticipated future income in (10), while that is impossible in (3).

It is worth noting that general equilibrium assumptions are imposed as part of the derivation of (10): in particular, the path $\{r_s\}_{s=0}^{\infty}$ is not included as an argument, because of our assumption that monetary policy fixes $r_s = r$. This distinguishes (10) from intertemporal consumption functions as defined in Kaplan et al. (2018) and Farhi and Werning (2019), which summarize the household problem without imposing further equilibrium restrictions. We generalize $\mathcal{C}_t$ and move closer to this other approach in section 2.5.

### 2.3 The intertemporal Keynesian cross

To solve for general equilibrium, fiscal policy and market clearing conditions remain to be specified. We let the government exogenously set sequences of taxes $T_t$ and spending $G_t$, issuing bonds to satisfy its budget constraint, $B_t = (1+r)\,B_{t-1} + G_t - T_t$, and keeping $B_t$ bounded. The goods market clearing condition is $C_t + G_t = Y_t$. Combined with equation (10) and the definition of $Z_t$, we obtain:

$$\mathcal{C}_t\left(\{Y_s - T_s\}\right) + G_t = Y_t \tag{11}$$

Just like (4), this equation captures a fixed point in output. However, the fixed point in (11) involves infinite-dimensional sequences: given fiscal policy $\{G_t\}$ and $\{T_t\}$, (11) can be solved for $\{Y_t\}$ and $\{C_t\}$. We have derived an intertemporal analog to the nonlinear version of the static Keynesian cross (4).

Equation (11) implies that all real aggregates in the economy, $\{Y_t, C_t, G_t, T_t, B_t, r_t\}$, are determined without any reference to nominal quantities. To obtain the nominal wage and price levels $W_t = P_t$, as well as the nominal interest rate $i_t$, we need to specify the wage Phillips curve—the dynamic relation between wage inflation $\pi_t^w$ and real aggregates. In appendix A.3, we derive such a Phillips curve for our environment by generalizing the standard microfoundation in the sticky-wage New Keynesian literature (e.g. Erceg, Henderson and Levin 2000 and Schmitt-Grohé and Uribe, 2005) to models with household heterogeneity.

**Linearization.** Just as we linearized the nonlinear static Keynesian cross (4), we next linearize (11). To do so, a few technical assumptions are required. We maintain the assumption that the

[10] From now on, we drop the $s = 0 \ldots \infty$ sub- and superscripts on the arguments of $\mathcal{C}$.

economy is initially at a steady state with constant $Y, G, T$, and $r > 0$. We consider bounded perturbations in output $d\mathbf{Y} \equiv \{dY_t\} \in \ell^\infty$ and similarly bounded policies $d\mathbf{T}, d\mathbf{G} \in \ell^\infty$.[11] We assume that the function $\mathcal{C} : \ell^\infty \to \ell^\infty$ is Fréchet-differentiable around the steady state, in other words that its derivative, a *sequence-space Jacobian* (Auclert et al. 2021a), is a bounded linear operator $\mathbf{M} : \ell^\infty \to \ell^\infty$.

For all models introduced in section 4, this operator can be represented by an infinite matrix $[M_{ts}]_{t,s=0}^\infty$, meaning that for any $\mathbf{x} = \{x_0, x_1, \ldots\} \in \ell^\infty$, if $\mathbf{y} = \mathbf{M}\mathbf{x} = \{y_0, y_1, \ldots\} \in \ell^\infty$, then $y_t$ can be written as $y_t = \sum_{s=0}^\infty M_{ts} x_s$ (see appendix A.4). We show this directly for our analytical models by explicitly deriving the matrix, and we verify this numerically for our quantitative models. Going beyond the models of section 4, we also provide a simple condition in appendix A.4 that ensures a matrix representation. Henceforth in this paper we will assume that such a representation exists, and for notational convenience we will use $\mathbf{M}$ interchangeably to denote the operator and the infinite matrix representing it.[12]

We refer to $\mathbf{M}$ as the matrix of *intertemporal MPCs*, or *iMPCs* for short. Entry $M_{ts}$ of $\mathbf{M}$ gives the aggregate consumption response at date $t$ to an anticipated increase in aggregate after-tax income at date $s$. Entry $M_{00}$ is the impact response to an unanticipated increase in income, which is usually how the static $mpc$ is estimated—making intertemporal MPCs $\mathbf{M}$ a generalization of the $mpc$.

We consider the first-order perturbation solution to (4) for a bounded fiscal policy shock $d\mathbf{G}$, $d\mathbf{T}$ satisfying the intertemporal budget constraint of the government,

$$\sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{dT_t}{(1+r)^t} \tag{12}$$

We next characterize the sequences $d\mathbf{Y}$ that solve (11) to first order.

**Proposition 1** (The intertemporal Keynesian cross)**.** *Consider a bounded shock $d\mathbf{G}, d\mathbf{T}$ satisfying (12). Then, any impulse response of output, $d\mathbf{Y}$, must satisfy:*

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T} + \mathbf{M} \cdot d\mathbf{Y} \tag{13}$$

*where $\mathbf{M}$ has entries $M_{ts} \equiv \frac{\partial \mathcal{C}_t}{\partial Z_s}$, and satisfies $\sum_{t=0}^\infty \frac{M_{ts}}{(1+r)^{t-s}} = 1$ for all $s$.*

Equation (13) is the *intertemporal Keynesian cross*, or *IKC* for short. It is the intertemporal analog to the static Keynesian cross (5). In the IKC, the impulse response of output $d\mathbf{Y}$ depends on the entire time path of government spending $d\mathbf{G}$ and taxes $d\mathbf{T}$. For example, as consumption today typically responds to past and future after-tax income in (10), output today also generally depends on taxes and government spending in the past and future.

[11] Here boldface denotes an infinite-dimensional sequence indexed by $0, 1, \ldots$, which we represent as a column vector, and $\ell^\infty$ is the space of bounded sequences endowed with the sup norm.

[12] More generally, it is possible that a bounded linear operator $\mathbf{M} : \ell^\infty \to \ell^\infty$ may not have an infinite matrix representation: indeed, we describe a non-economic counterexample in appendix A.4. As we discuss there, it seems unlikely that such a counterexample could emerge from a reasonable economic model.

Analogously to the static Keynesian cross, where $mpc$ is the sole determinant of the output response to fiscal policy, in the intertemporal Keynesian cross, the matrix of iMPCs $\mathbf{M}$ is the sufficient statistic for this response. This result puts $\mathbf{M}$ at the heart of this paper. In section 3, we discuss how much we can learn about $\mathbf{M}$ from the data alone. In section 4, we derive $\mathbf{M}$ for several models of the intertemporal consumption function $\mathcal{C}(\cdot)$, and explain how the data disciplines the structural parameters of these models. Sections 5–7 investigate the role of $\mathbf{M}$ in determining the effects of fiscal policy across our models.

In the next subsection, we start by solving the IKC (13) for a given $\mathbf{M}$.

### 2.4 Solving the intertemporal Keynesian cross

In the static Keynesian cross, provided that $mpc < 1$, the solution to (5) is always given by (6), and can also be obtained from the iteration (7). For the intertemporal Keynesian cross (13), it is tempting to proceed analogously, writing $d\mathbf{Y} = (\mathbf{I} - \mathbf{M})^{-1}(d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T})$ where $\mathbf{I}$ is the identity, or $d\mathbf{Y} = (\mathbf{I} + \mathbf{M} + \mathbf{M}^2 + \ldots)(d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T})$.

Unfortunately, the solution is not so straightforward. The reason is that, in a present value sense, intertemporal MPCs must aggregate to 1—the present value of any column $s$ of $\mathbf{M}$ is identical to the present value of a date-$s$ income transfer:

$$\sum_{t=0}^{\infty} \frac{M_{ts}}{(1+r)^t} = \frac{1}{(1+r)^s} \tag{14}$$

We can write this more succinctly by introducing the vector of present-value discount factors as $\mathbf{q} \equiv \left\{ \left(\frac{1}{1+r}\right)^t \right\}_{t=0}^{\infty}$. Then (14) reads $\mathbf{q}'\mathbf{M} = \mathbf{q}'$, or alternatively $\mathbf{q}'(\mathbf{I} - \mathbf{M}) = 0$. It follows that the inverse $(\mathbf{I} - \mathbf{M})^{-1}$ cannot exist: no sequence with nonzero present value lies in the range of $\mathbf{I} - \mathbf{M}$. Similarly, the series $\mathbf{I} + \mathbf{M} + \mathbf{M}^2 + \ldots$ may diverge.

However, a bounded solution to (13) can still exist. This is because the term $d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T}$ on the right hand side of the IKC (13) does, in fact, have a present value of zero: $\mathbf{q}'d\mathbf{G} - \mathbf{q}'\mathbf{M} \cdot d\mathbf{T} = \mathbf{q}'d\mathbf{G} - \mathbf{q}'d\mathbf{T}$, which is zero by the government's intertemporal budget constraint (12). The term $d\mathbf{G} - \mathbf{M} \cdot d\mathbf{T}$ can, in fact, lie in the range of $\mathbf{I} - \mathbf{M}$.[13]

To obtain the solution when it exists, we first pre-multiply the IKC (13) with an appropriately chosen infinite matrix $\mathbf{K}$:

$$\mathbf{K}(\mathbf{I} - \mathbf{M})d\mathbf{Y} = \mathbf{K}(d\mathbf{G} - \mathbf{M}d\mathbf{T}) \tag{15}$$

We then observe that, if $\mathbf{K}(\mathbf{I} - \mathbf{M})$ is invertible, the solution is simply given by $d\mathbf{Y} = (\mathbf{K}(\mathbf{I} - \mathbf{M}))^{-1}\mathbf{K}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$. Our next proposition shows that there is in fact a choice of $\mathbf{K}$ such that this strategy works whenever the IKC has a unique solution.[14] To state the proposition, we denote

[13] Loosely speaking, in (6), the denominator is effectively zero but the numerator is also zero, so that it is possible to have a finite solution $dY_t$.

[14] Note that, in finite dimensions, this approach never works: no matter how we pre-multiply a finite-dimensional singular square matrix, it remains singular. In infinite dimensions, however, this is possible. A simple example is that neither the lead operator $\mathbf{F}$ nor its transpose the lag operator $\mathbf{L}$ is invertible, but their product $\mathbf{FL} = \mathbf{I}$ is.

by $\mathbf{F}$ the lead operator that maps $\{x_0, x_1, \ldots\}$ to $\{x_1, x_2, \ldots\}$, corresponding to a matrix with ones directly above the diagonal. We have:

**Proposition 2.** *Let* $\mathbf{K} \equiv -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^t$. *There exists a unique solution* $d\mathbf{Y} \in \ell^\infty$ *to (13) for all shocks* $d\mathbf{G}, d\mathbf{T} \in \ell^\infty$ *satisfying (12) if and only if* $\mathbf{K}(\mathbf{I}-\mathbf{M})$ *is invertible. When this is the case, the solution is given by*

$$d\mathbf{Y} = \mathcal{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T}) \tag{16}$$

*where the multiplier* $\mathcal{M}$ *is the bounded linear operator defined by* $\mathcal{M} \equiv (\mathbf{K}(\mathbf{I}-\mathbf{M}))^{-1}\mathbf{K}$. *This multiplier satisfies* $\mathcal{M}(\mathbf{I}-\mathbf{M}) = \mathbf{I}$ *as well as* $(\mathbf{I}-\mathbf{M})\,\mathcal{M}\mathbf{x} = \mathbf{x}$ *for all* $\mathbf{x} \in \ell^\infty$ *such that* $\mathbf{q}'\mathbf{x} = 0$.

The proposition, proved in appendix A.6, picks $\mathbf{K}$ such that $\mathbf{K}(\mathbf{I}-\mathbf{M})$ is the *asset Jacobian* $\mathbf{A}$—the derivative of the aggregate asset function $\mathcal{A}_t(\{Z_s\})$ defined in the previous section. With this choice of $\mathbf{K}$, we also have $\mathbf{K}(d\mathbf{G} - d\mathbf{T}) = d\mathbf{B}$, where $dB_t$ is the path of debt implemented by the government. Hence, (15) reads $\mathbf{A}(d\mathbf{Y} - d\mathbf{T}) = d\mathbf{B}$, which is the linearized asset market clearing condition. By Walras's law, if we can find such a solution that clears the asset market, then it also will clear the goods market at all dates. Interestingly, even though the shock to demand from fiscal policy, $d\mathbf{G} - \mathbf{M}d\mathbf{T}$, has zero present value, and even though $\mathbf{M}$ conserves present value, the general equilibrium output response (16) generally does *not* have zero present value.

How do we know when $\mathbf{A} = \mathbf{K}(\mathbf{I}-\mathbf{M})$ is invertible? We pursue two routes in this paper. For simple models, $\mathbf{A}$ has an analytical form, and we will check invertibility directly. For quantitative models, where $\mathbf{A}$ is constructed numerically, we check that the winding number of $\mathbf{A}$ is zero, applying the criterion in Auclert, Rognlie and Straub (2023b). We discuss the numerical implementation of proposition 2 in appendix A.7.

### 2.5 Extending the intertemporal Keynesian cross

We just derived an intertemporal Keynesian cross (13) for the response to fiscal policy shocks. We now discuss how this equation extends to other shocks and to more general environments.

To do this, we generalize the intertemporal consumption function (10). We allow for (a) a general allocation rule, $n_{it} = \mathcal{N}(e_{it}, N_t)$, subject to $\int \mathcal{N}(e_{it}, N_t)di = N_t$ at all times; (b) a general retention function $z_{it} = \mathcal{Z}(e_{it}n_{it}, T_t)$; (c) real interest rates $r_t$ to vary; and (d) a general shifter $\Theta$ to the consumption function. As we show in appendix A.8, the consumption function then becomes:

$$C_t = \mathcal{C}_t(\{Y_s, T_s, r_s, \Theta\})$$

and the nonlinear fixed-point equation for output is now:

$$\mathcal{C}_t(\{Y_s, T_s, r_s, \Theta\}) + G_t = Y_t \tag{17}$$

The shifter $\Theta$ can represent any shock that shuffles intertemporal consumption out of income, while leaving the aggregate budget constraint undisturbed: for instance, a shock to preferences, borrowing constraints, income risk, or inequality. By having $Y_t$ and $T_t$ enter separately, we allow changes in labor income and taxes to have different incidence across agents—for instance, taxes may be increased mostly on the rich, but higher aggregate income benefits the poor as well. Additionally, monetary policy now implements an arbitrary exogenous real interest rate path $\{r_t\}$.[15] With slight abuse of notation, we define the vector $d\mathbf{r}$ to have elements $d\log(1+r_t) = \frac{dr_t}{1+r}$. To respect the intertemporal government budget constraint, fiscal policy shocks must now satisfy $\mathbf{q}'d\mathbf{T} = \mathbf{q}'d\mathbf{G} + B\mathbf{q}'d\mathbf{r}$.

Following the same steps as in proposition 1, we can differentiate (17) to obtain:

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + \partial\mathbf{C} + d\mathbf{G} - \mathbf{M}^T d\mathbf{T} + \mathbf{M}d\mathbf{Y} \tag{18}$$

where $\partial\mathbf{C} \equiv \frac{\partial\mathcal{C}}{\partial\Theta}d\Theta$ is defined to be the direct effect of the shifter on consumption, and now $\mathbf{M} \equiv \frac{\partial\mathcal{C}}{\partial Y}$, $\mathbf{M}^T \equiv \frac{\partial\mathcal{C}}{\partial(-T)}$, and $\mathbf{M}^r \equiv \frac{\partial\mathcal{C}}{\partial\log(1+r)}$. If $Y_t$ has the same incidence as $T_t$ for all agents, so that individual income is determined by $Y_t - T_t$, we have $\mathbf{M}^T = \mathbf{M}$.[16]

Equation (18) has the same exact form as (13), except that the demand impulse $d\mathbf{G} - \mathbf{M}d\mathbf{T}$ is replaced by the more general expression $\mathbf{M}^r d\mathbf{r} + \partial\mathbf{C} + d\mathbf{G} - \mathbf{M}^T d\mathbf{T}$; as we show in appendix A.8, this sequence still has zero present value. It immediately follows that the solution to (18) is:

$$d\mathbf{Y} = \mathcal{M}\left(\mathbf{M}^r d\mathbf{r} + \partial\mathbf{C} + d\mathbf{G} - \mathbf{M}^T d\mathbf{T}\right) \tag{19}$$

Equation (19) shows that many different kinds of shocks—not only fiscal shocks, but also shocks to interest rates, preferences, borrowing constraints, and inequality—work through the same general equilibrium mechanisms, governed by iMPCs $\mathbf{M}$ and the resulting multiplier operator $\mathcal{M}$. Indeed, $\mathcal{M}$ allows us to make predictions about transmission from partial to general equilibrium more generally. For instance, if a deleveraging shock has a direct consumption effect of $\partial\mathbf{C}$, then a fiscal shock that perturbs government spending by $d\mathbf{G} = \partial\mathbf{C}$, leaving taxes unaffected, will have exactly the same general equilibrium output effect, because d$\mathbf{G}$ and $\partial\mathbf{C}$ both have the same multiplier $\mathcal{M}$ in (19). The same result applies to shocks to inequality (Auclert and Rognlie 2018), taxes (Wolf 2023b), and monetary policy (Wolf 2023a).[17]

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[15] See appendix A.2 for details on how a monetary authority controlling nominal interest rates can implement any arbitrary path of real interest rates in this economy.

[16] In the absence of $d\mathbf{r}$ and $\partial\mathbf{C}$ shocks, in this case we recover the original form of the intertemporal Keynesian cross (13) under slightly more general assumptions: it is not necessary to assume proportional labor allocation and our specific tax rule, only that individual income is determined by $Y_t - T_t$.

[17] Since two models with different $\mathbf{M}$ matrices have different $\mathcal{M}$, equation (19) shows that they have a different impulse responses of output to pure government spending shocks $d\mathbf{G}$. In the language of Kaplan and Violante (2018), they are nonequivalent for these shocks. However, they could be weakly equivalent, i.e. deliver the same impulse responses, for monetary policy (see footnote 41), or even strongly equivalent for balanced-budget fiscal policy (see proposition 3, where after-tax income entering the household problem is unchanged in equilibrium).

### 2.6 Limitations of the intertemporal Keynesian cross

In more general environments, the intertemporal Keynesian cross no longer holds with our definition of the $\mathbf{M}$ matrix. This is typically because endogenous variables other than aggregate output matter for aggregate consumption.

For instance, suppose that nominal interest rates are set according to a Taylor rule. Then, the real interest rate path is determined by inflation, which is itself endogenous, and this matters for consumption. In this case, even our generalized intertemporal Keynesian cross (18) cannot be used to solve directly for output, and must instead be solved jointly with another equation that characterizes the real interest rate path $d\mathbf{r}$. As we show in appendix B.1, it is still possible to reduce equilibrium to a single equation with the same form as the IKC, by replacing the $\mathbf{M}$ with a more complex $\tilde{\mathbf{M}}$ that reflects the feedback through endogenous $d\mathbf{r}$. But now, constructing $\tilde{\mathbf{M}}$ requires knowledge of structural parameters beyond just iMPCs out of income.

Appendix B shows that a variety of other changes to the model—nominal bonds, sticky prices, endogenous labor supply with GHH preferences, durable goods, and investment—have similar implications: they break the original IKC, and replace $\mathbf{M}$ with a more complicated $\tilde{\mathbf{M}}$ that involves other structural parameters. Although this representation can be useful, $\tilde{\mathbf{M}}$ is harder to interpret than $\mathbf{M}$ and lacks a simple sufficient statistic interpretation. This representation also requires the ability to reduce equilibrium to a sequence of equilibrium conditions in a single goods market.[18]

Away from these cases, it is generally no longer possible to obtain a single equation mapping shocks to general equilibrium outcomes. However, it is often still possible to summarize the aggregate consumption behavior in the model using our original definition of $\mathbf{M}$, combined with other derivatives of the consumption function. These still constitute sufficient statistics for how household consumption responds to variables such as income and interest rates, but now the equilibrium evolution of these variables depends on the non-household parts of the model as well. Section 6 takes this approach, finding that, in the quantiative environment we consider, the only additional piece of information beyond $\mathbf{M}$ required to summarize aggregate household behavior is the impulse response of consumption to capital gains.

## 3 Empirical evidence on intertemporal MPCs

In the intertemporal Keynesian cross, the matrix of iMPCs $\mathbf{M}$ is a sufficient statistic for the output response to fiscal policy. In this section, we explore what we can learn about $\mathbf{M}$ from micro data on consumption responses to income changes.

[18] This is not generally true, for instance, when there is flexible labor supply and preferences are not GHH, since then two endogenous prices (real interest rates and wages) must clear two markets (goods and labor).

### 3.1 Response to unexpected income shocks

To estimate the first column of $\mathbf{M}$, we observe that given the assumptions of section 2.2, this column can be expressed as an average of individual responses to an unexpected income shock, $\partial \mathbb{E}_0 [c_{it}] / \partial z_{i0}$, weighted by after-tax income in the year of the income shock. Formally, we have:[19]

**Lemma 1.** *The first column of* $\mathbf{M}$ *can be written as:*

$$M_{t0} = \int \frac{z_{i0}}{\int z_{\imath 0} d\imath} \cdot \frac{\partial \mathbb{E}_0 [c_{it}]}{\partial z_{i0}} di \tag{20}$$

We propose two sources of evidence for the path of individual responses $\partial c_{it} / \partial z_{i0}$.[20] In both cases, consumption $c_{it}$ is defined as spending on all goods, including durable goods, consistent with the theory (see appendix B.5).

**Norwegian lottery evidence.** Our first source of evidence comes from Norwegian administrative data, as analyzed in Fagereng et al. (2021). The data includes comprehensive information on consumption and uses the random winnings of lotteries to identify the dynamic consumption responses to income shocks. The authors' main estimating equation is:

$$c_{i,t+k} = \alpha_i + \tau_{t+k} + \gamma_k \text{lottery}_{it} + \delta X_{it} + \varepsilon_{it} \qquad k = 0, \dots 5 \tag{21}$$

where $c_{i,t+k}$ is consumption of individual $i$ in year $t+k$, $\alpha_i$ an individual fixed effect, $\tau_{t+k}$ a time fixed effect, $X_{it}$ are household characteristics, and $\text{lottery}_{it}$ is the amount household $i$ wins in year $t$. The authors provided us with regression results weighted by after-tax incomes at the time of the lottery win.[21] Since lottery wins are not forecastable and disbursed at the time they are announced, the estimated $\widehat{\gamma_k}$ correspond exactly to the weighted average in (20), and therefore to the $M_{k0}$ that matters for the theory.

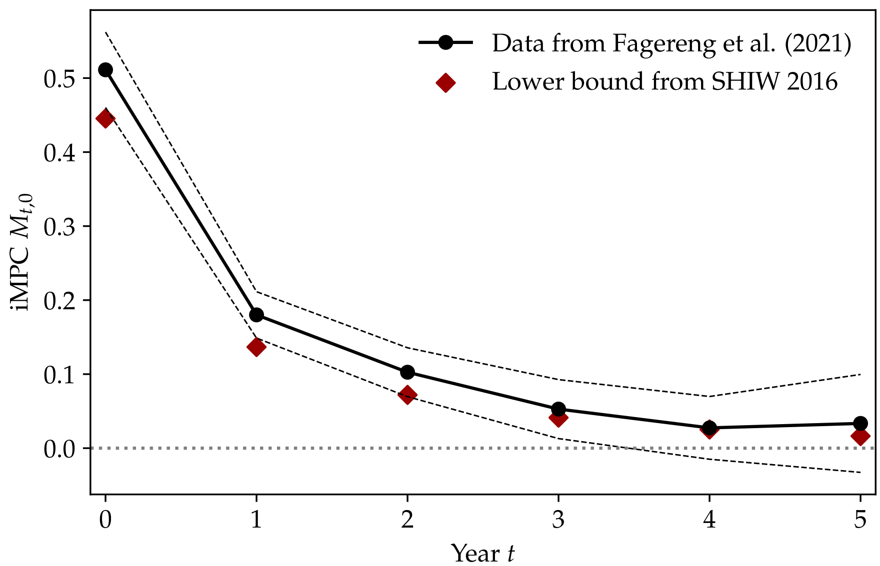
The black dots in figure 1 represent the point estimates for $\widehat{\gamma_0}$ through $\widehat{\gamma_5}$, together with 99% confidence intervals. Consistent with a large empirical literature, the annual MPC out of a one-time transfer is large, at about 0.51. What the literature has not stressed as much, but clearly appears in the Norwegian data, is that the iMPC in the year following the transfer is also quite large, at around 0.18. After this point, the iMPCs slowly decay and become statistically insignificant around year 4.

[19] For a proof, see appendix C.1. When the incidence of taxes and labor income are different, this approach can be generalized to separately estimate $\mathbf{M}^T$, using a different weighting function.

[20] The existing literature mostly focuses on estimating contemporaneous marginal propensities to consume (which is helpful to inform $M_{00}$ in our notation), e.g. Shapiro and Slemrod (2003), Johnson, Parker and Souleles (2006), Blundell, Pistaferri and Preston (2008), Jappelli and Pistaferri (2014), and Fuster, Kaplan and Zafar (2021).

[21] Our reference estimates are their weighted full sample estimates, including responses to all sizes of lottery winnings up to \$150,000 (see appendix C.2 for additional details.) An alternative would have been to restrict the sample to only small winnings. However, MPC estimates in this sample are inherently imprecisely estimated due to the large noise-signal ratio.

Figure 1: iMPCs in the Norwegian and Italian data




**A lower bound from Italian survey evidence.** Our second source of evidence is a lower bound estimate for $\partial \mathbb{E}_0 [c_{it}] / \partial z_{i0}$ constructed from survey data on MPCs. We implement this bound using the 2016 version of the Italian Survey of Household Income and Wealth (SHIW), which asks survey respondents to report their annual contemporaneous MPC, $MPC_i \equiv \partial c_{i0} / \partial z_{i0}$.

We obtain a point estimate for $M_{00}$ by weighting $MPC_i$ by income. To bound $M_{t0}$ for $t > 0$, we propose the following idea. In any model where current income and assets enter the budget constraint interchangeably, $MPC_i$ also gives consumption at the margin out of saved assets.[22] Now consider $M_{10}$. How small can this be, assuming the cross-sectional distribution of $MPC_i$ is the same in years 0 and 1? It is smallest when the households who save the most out of an income shock in year 0—i.e. the households with the lowest $MPC_i$, and the highest savings $(1 + r) \cdot (1 - MPC_i)$ entering the next period—are again the households who have the lowest $MPC_i$ out of their savings in year 1. It follows that a weighted average of $(1 + r) \cdot (1 - MPC_i) \cdot MPC_i$ gives a lower bound $\underline{M_{10}}$ for $M_{10}$.

In appendix C.3, we formalize this argument, and extend it to all iMPCs $M_{t0}$ for $t > 0$.[23] For $t > 1$, the implied lower bounds are on the cumulative iMPC—the present value of spending through each date $t$—and we calculate $\underline{M_{t0}}$ that sum to these cumulative bounds.

The red diamonds in figure 1 display our $\underline{M_{t0}}$. The results are remarkably consistent with

[22] This is true for all models in section 4 except the two-account HA model, where assets in the illiquid account do not enter interchangeably with income, and therefore the argument for the lower bound does not apply. We will use the lower bound to reject other models where it does apply.

[23] To deal with weighting by income at date 0, the full argument in appendix C.3 requires an intuitive additional assumption on the dynamic relationship between date-0 income and subsequent MPCs. We verify that this assumption holds in this paper's one-account heterogeneous-agent model. We also validate that the overall distribution of MPCs is stationary by comparing the 2010 and 2016 distributions of MPCs in the SHIW.

those obtained from the Norwegian administrative data. While the weighted contemporaneous MPC is slightly lower, at 0.44, the subsequent lower bound estimates are closely aligned with those obtained from the Norwegian data. The year-1 lower bound, in particular, is equal to 0.14 and thus only slightly below the Norwegian estimate of 0.18. Recall that this bound is a weighted average of $(1+r) \cdot (1 - MPC_i) \cdot MPC_i$, so it is entirely accounted for by individuals in the sample that report *intermediate* MPCs, not too close to either 0 or 1. This suggests that matching the iMPCs in the data will require models that generate an entire distribution of MPCs, including intermediate MPCs, strictly between zero and one.

### 3.2 Other evidence on iMPCs

We now discuss what we know from the data about the other elements of $\mathbf{M}$, beyond the first six elements of its first column.

**Other elements of the first column of M.** Proposition 1 restricts the first column of $\mathbf{M}$ to have a present discounted value of 1. In the Norwegian data, the present value $\sum_{k=0}^{5} \frac{\widehat{\gamma_k}}{(1+r)^k}$ is below, but not too far from, 1 for reasonable values of $r$: for instance at $r = 5\%$ we obtain a present value of 0.87. This suggests that, while households in the aggregate spend the majority of lottery earnings in the first years after receipt, they also save a small fraction, which then remains available to be spent in later years. Figure 2 in Fagereng et al. (2021) confirms that this is the case: the point estimate for the increase in total assets by year 5 is 0.16. Around 50% of this total is accounted for by stocks, bonds and mutual funds. This suggests that some of these savings may be held for the long term, and that households may spend them down fairly slowly.

**Other columns of M: expected income shocks.** In an ideal world, we would also have information about the other columns of the $\mathbf{M}$ matrix. Unfortunately, there currently exists very limited information on consumption responses to anticipated changes in income one year out or later. As we discuss in appendix C.4, the existing evidence points to the presence of some, albeit modest, anticipation effects (e.g. Fuster et al. 2021, Agarwal and Qian 2014, Di Maggio et al. 2017). The evidence is currently too imprecise for us to confidently use it as a model input.

**Takeaway: the need for additional structure.** The discussion in this section suggests that existing data are currently too limited to allow us to fully construct $\mathbf{M}$ without imposing more structure from a model. We thus proceed by specifying several microfounded models of consumption and saving, each of which implies an iMPC matrix $\mathbf{M}$. We show how the existing data on $\mathbf{M}$ can be used to discriminate between the models, and how each model's $\mathbf{M}$ shapes its fiscal policy implications. As the empirical literature estimating iMPCs develops further, we should be able to learn more about $\mathbf{M}$ and refine our view of the types of models that can fit the empirical evidence.

## 4 Models of iMPCs and their fit to the data

We now specify several models of the intertemporal consumption function $\mathcal{C}_t\left(\{Z_s\}_{s=0}^{\infty}\right)$, starting from standard models with analytical solutions and proceeding to increasingly complex quantitative models. Each of the models we study offers a low-dimensional parameterization of **M**, which we compare and calibrate to the evidence in the preceding section.

### 4.1 Analytical models: RA, BU, TA and TABU

We start by describing a class of four analytically tractable models, for which **M** admits a closed-form solution. This section summarizes our results; appendix D.2 provides detailed derivations.

There is a mass $1-\mu$ of unconstrained agents. These agents earn post-tax income $Z_t \equiv Y_t - T_t$ in period $t$ and have access to a risk-free bond that pays the real interest rate $r$. Given initial bonds $a_{-1}^u$, they solve the problem:

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t^u\right) - v\left(n_t^u\right) + \chi\left(a_t^u\right) \right\} \\ \text{s.t.} \quad & c_t^u + a_t^u = Z_t + (1+r)\, a_{t-1}^u \end{aligned} \tag{22}$$

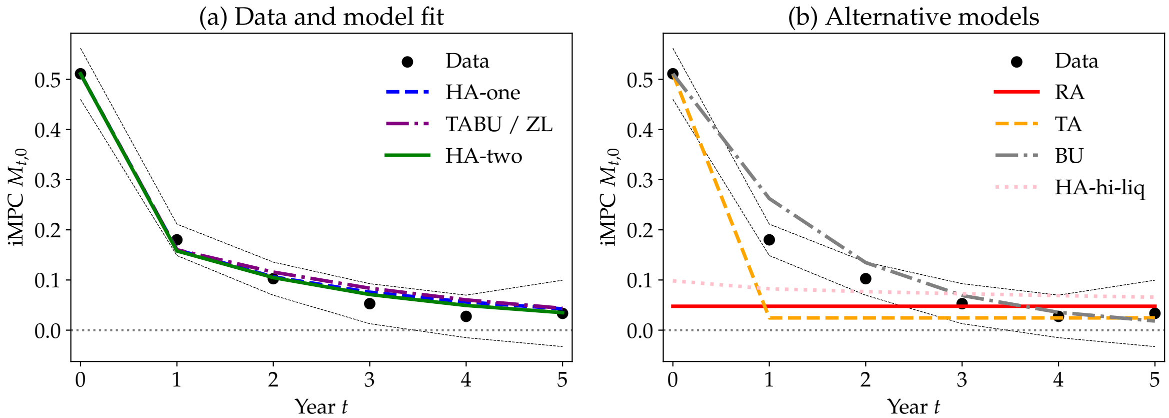
where $u$ (increasing and concave) is the utility from consumption, $v$ (increasing and convex) the disutility from labor, and $\chi$ (concave, though not necessarily increasing everywhere) is the utility for holding assets $a_t$. The remaining mass $\mu$ of agents are constrained, or "hand-to-mouth", with flow utility $u\left(c_t^c\right) - v(n_t^c)$, and consumption $c_t^c = Z_t$. Aggregate consumption is $C_t = (1-\mu)\, c_t^u + \mu c_t^c$. We study these models around a steady state with constant $Y, T, c^c, c^u$, and $a^u$.

Two canonical models are nested in this setting, as well as two more recent models of household behavior. The two canonical models are the *representative-agent model* (RA), which only has unconstrained agents and no assets in utility, $\mu = \chi = 0$; and the *two-agent model* (TA), which has some fraction $\mu > 0$ of constrained agents but no assets in utility, $\chi = 0$. The two more recent models are, first, the *asset-in-utility* or *bond-in-utility model* (BU) which only has unconstrained agents $\mu = 0$, but $\chi \neq 0$. And second, we introduce the *two-agent bond-in-utility model* (TABU), which has both $\mu > 0$ and $\chi \neq 0$. The steady-state assumption imposes that $\beta\,(1+r) = 1$ in both RA and TA. By contrast, BU and TABU can have any $\beta\,(1+r)$, depending on the sign of $\chi'\left(a^u\right)$.

**The two canonical models: RA and TA.** The RA model provides the simplest and earliest theory of the intertemporal consumption function (Friedman 1957). This model admits the **M** matrix:

$$\mathbf{M}^{RA} = \begin{pmatrix} (1-\beta) & (1-\beta)\,\beta & (1-\beta)\,\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\,\beta & (1-\beta)\,\beta^2 & \\ (1-\beta) & (1-\beta)\,\beta & (1-\beta)\,\beta^2 & \\ \vdots & & & \ddots \end{pmatrix} = (1-\beta)\,\mathbf{1}\mathbf{q}' = \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}} \tag{23}$$

Figure 2: iMPCs in the Norwegian data and in several models




*Notes*: All models are calibrated to match $r = 0.05$. RA does not have any other free parameter. The single free parameter in BU ($\lambda$), TA ($\mu$), HA-one ($A/Z$) and HA-two ($\nu$) is calibrated to match $M_{00} = 0.51$. The additional free parameter in TABU and ZL ($\mu$) is calibrated to match $M_{10} = 0.16$ (its value in the HA-one model). The HA-two and HA-hi-liq models are calibrated to an aggregate ratio of assets to post-tax income of $A/Z = 6.29$, its value in the model with capital in section 7.

where $\mathbf{1}$ is a vector of ones, and $\mathbf{q} \equiv \left\{ \left( \frac{1}{1+r} \right)^t \right\}_{t=0}^{\infty}$ is the vector of discount factors defined in section 2.4, and here $\frac{1}{1+r} = \beta$. $\mathbf{M}^{RA}$ encodes well-known permanent-income consumption behavior: each period, agents spend a constant fraction $1 - \beta$ out of the present value of their income. Observe that $\mathbf{M}^{RA}$ has rank 1, and is described by the single parameter $\beta$. Therefore, once we calibrate the model to a target real interest rate $r$, the RA model has no free parameters.

The red line in figure 2(b) shows the first column of $\mathbf{M}^{RA}$: the impulse response to an unexpected date-0 income shock. This impulse response is flat at $1 - \beta = \frac{r}{1+r}$, where we pick $r = 5\%$. Clearly, the RA model cannot fit the data for any value of $r$. As figure 3(a) shows, the other columns of $\mathbf{M}^{RA}$ are also flat, reflecting the ability of agents to borrow and smooth consumption perfectly in anticipation of any future increase in income.

A classic strategy to raise MPCs is to assume that a fraction $\mu$ of agents are hand-to-mouth: this is the TA model, also known as the spender-saver model (Campbell and Mankiw 1989). Since for hand-to-mouth households as a group, $\mathbf{M}$ equals the identity $\mathbf{I}$ (agents consume income in the period they receive it), the $\mathbf{M}$ matrix of the TA model as a whole is the weighted average:

$$\mathbf{M}^{TA} = (1 - \mu)\,\mathbf{M} + \mu \mathbf{I} \tag{24}$$

Relative to the RA model, $\mathbf{M}^{TA}$ has one additional free parameter $\mu$. This parameter can be calibrated to generate any desired $M_{00}$. The orange line in figure 2(b) shows the first column when we calibrate $\mu$ to match $M_{00}$ in the Norwegian data. The iMPCs drop off immediately after the receipt of income, and therefore cannot match the data's $M_{10}$. As figure 3(b) shows, other columns have the same feature: the $\mathbf{M}^{TA}$ matrix features sharp spikes when income is received, representing the response of constrained hand-to-mouth agents in that period, and that period alone.

**Recent tractable models: BU and TABU.** A simple model that relaxes this sharp spike property, yet maintains high MPCs, is the BU model, as recently studied by Kaplan and Violante (2018), Hagedorn (2018) and Michaillat and Saez (2021). The $\mathbf{M}$ matrix of the BU model has three parameters: $\beta$ and $r$, as well as $\lambda$, the slope of the asset policy function in the steady-state. In appendix D.2, we provide a complete analytical expression for $\mathbf{M}^{BU}$ as a function of these parameters. In particular, we show that the first column of $\mathbf{M}^{BU}$ decays exponentially:[24]

$$M_{t0}^{BU} = \left(1 - \frac{\lambda}{1+r}\right) \cdot \lambda^t \qquad \text{for all } t \geq 0 \tag{25}$$

We keep $r = 5\%$ and calibrate $\lambda$ to match $M_{00}$ from the Norwegian data in figure 2(b). As the figure shows, $M_{10}$ is significantly higher for the BU model than for the TA model. In fact, due to the exponential decay, $M_{10}^{BU} = \lambda M_{00}^{BU}$ is too large relative to the data.

Figure 3(c) plots other columns of $\mathbf{M}^{BU}$ for a choice of $\beta = 0.87$, the discount factor in the one-account heterogeneous-agent model introduced below. Observe that the columns $s$ of $\mathbf{M}^{BU}$ initially differ in shape, because for small $s$, agents have little or no time to spend in anticipation of the date-$s$ income shock. As we increase $s$, this effect goes away and the columns converge to a stable long-run pattern, which we derive analytically in the appendix. The existence of this long-run pattern is a property shared among all the remaining models we introduce in this section (see figures 3(d)–(f)).

Finally, by assuming $\mu$ hand-to-mouth and $1-\mu$ bond-in-utility households, we obtain the "TABU" model. This model offsets the disadvantages of the TA model, where $M_{10}$ was too low, and the BU model, where $M_{10}$ was too high. Its iMPC matrix is a weighted average of $\mathbf{I}$ and $\mathbf{M}^{BU}$, $\mathbf{M}^{TABU} = \mu \mathbf{I} + (1-\mu)\mathbf{M}^{BU}$. The TABU model has four parameters $r, \beta, \mu$ and $\lambda$. In figures 2(a) and 3(d), we leave $r$ and $\beta$ as before and calibrate $\mu$ and $\lambda$ to jointly match $M_{00}$ and $M_{10}$.[25]

### 4.2 The one-account heterogeneous-agent model and its zero liquidity limit

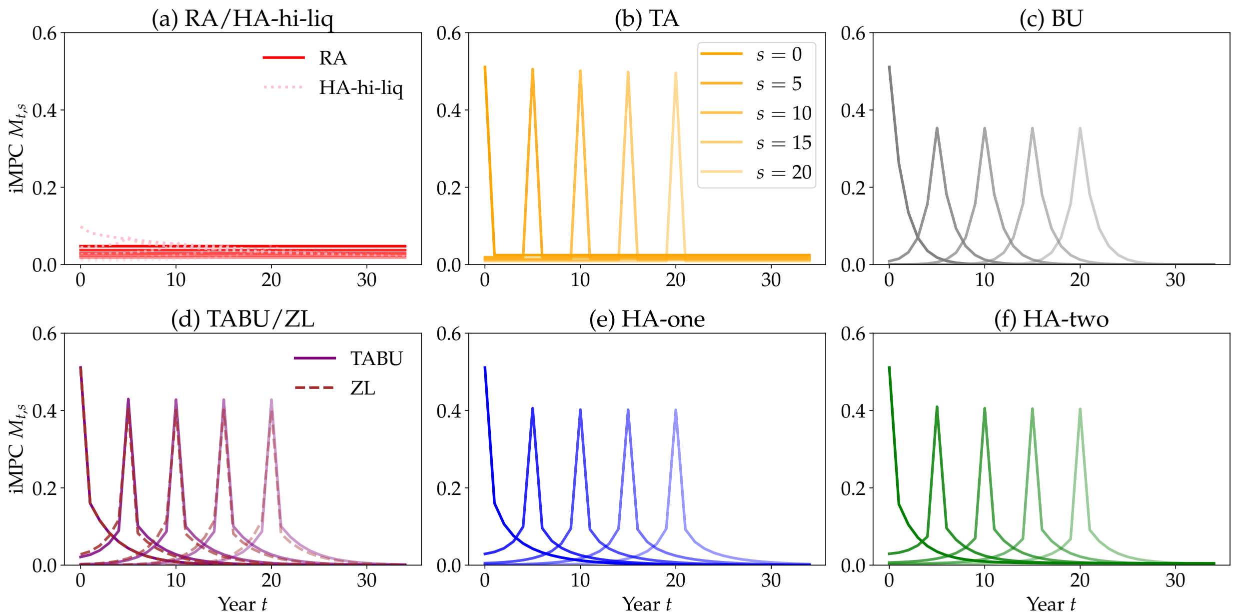
We now introduce the consumption function for the canonical one-account heterogenous-agent model, which is the backbone of a large literature following Bewley (1980).

The economy is populated by a unit mass of households, who face idiosyncratic income uncertainty. Agents vary in their idiosyncratic ability state $e_{it}$, which follows a Markov process with fixed transition matrix $\Pi$. The mass of agents in idiosyncratic state $e$ is always equal to $\pi(e)$, the probability of $e$ in the stationary distribution of $\Pi$. The average ability level is normalized to be one, so that $\sum_e \pi(e)\, e = 1$. As in section 2.2, agent $i$'s post-tax labor income is given by

[24] For other derivations of some of the iMPCs of a BU model, see Cantore and Freund (2021), Aggarwal, Auclert, Rognlie and Straub (2023), and Wolf (2023a). The former argues that this model is first-order equivalent to a model with portfolio adjustment costs. The latter two argue that it is also first-order equivalent to a perpetual-youth OLG model as in Blanchard (1985). See proposition 7 in Aggarwal et al. (2023) for a proof of this equivalence.

[25] $\beta = 0.87$ and $M_{10} = 0.16$ are chosen to be the same as in the next section's HA-one model to make these models comparable. Appendix D.2 provides analytical formulas showing the effect of changing these parameters for the $\mathbf{M}$ matrix, and section 5 provides related formulas for fiscal multipliers.

Figure 3: iMPCs in eight standard models




*Notes*: The models are calibrated as in figure 1. The discount factors, which are relevant for anticipation, are reported in table 2.

$z_{it} = \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di} Z_t$. Agent $i$ can only hold assets in a single, liquid account $a_{it}$. Given his initial asset position $a_{i,-1}$, his objective is to maximize utility:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left\{u\left(c_{it}\right) - v\left(N_t\right)\right\}\right] \tag{26}$$

subject to the budget constraint in (9) and to the borrowing constraint $a_{it} \geq 0$. Here, $u(c)$ is the constant elasticity of substitution (CES) utility function $\frac{c^{1-1/\sigma}}{1-1/\sigma}$ with intertemporal elasticity $\sigma$.

Given a calibration for the income process $(\Pi, \mathbf{e}, \theta)$ and $(r, \beta)$, the model generates a stationary distribution over its state variables $(a, e)$ and an aggregate consumption function of the form (10). We follow Heathcote et al. (2017) and set the curvature parameter in the retention function to $\theta = 0.181$. For our calibration of $\Pi$, we follow standard practice in the literature and assume that gross income follows an AR(1) process. We use Floden and Lindé (2001)'s estimates of the persistence of the US wage process, equal to 0.91 yearly, set the variance of innovations to match the standard deviation of log gross earnings in the US of 0.92 as in Auclert and Rognlie (2018), and discretize this process as an 11-point Markov chain. Appendix D.3 discusses the model's solution.

The traditional way to complete the calibration of this one-account model is to select the discount factor $\beta$ to hit a target for total assets. We calibrate the *HA-hi-liq model*, for "high-liquidity", to hit a ratio of assets to after-tax income $A/Z = 6.29$, its value in our quantitative model of section 7. This is typical for a calibration where households can invest in capital as well as bonds.[26]

[26] We calibrate $A/Y \simeq 3$, and $Z/Y \simeq 0.5$ to be consistent with the US labor share and the average tax rate on labor. See section 7.1.

Table 2: Calibrating models of the intertemporal consumption function

| Parameter | Parameter | RA | HA-hi-liq | TA | BU | TABU | ZL | HA-one | HA-two |
|---|---|---|---|---|---|---|---|---|---|
| $\sigma$ | Elasticity of int. substitution | 1 | | | | (same across all models) | | | |
| $r$ | Real interest rate (annual) | 0.05 | | | | (same across all models) | | | |
| $A/Z$ | Assets to post-tax income | 6.29 | 6.29 | 6.29 | 6.29 | 6.29 | 0 | 0.21 | 6.29 |
| $\beta$ | Discount factor (annual) | 0.95 | 0.94 | 0.95 | 0.87 | 0.87 | 0.87 | 0.87 | 0.93 |
| $\mu$ | Effective share of HtM | 0 | 0.01 | 0.49 | 0 | 0.29 | 0.29 | 0.30 | 0.36 |
| $\lambda$ | Effective persistence of assets | | | | 0.51 | 0.72 | 0.72 | | |
| $m$ | Effective MPC of saver | | | | 0.51 | 0.31 | 0.31 | | |
| $(\rho_e, \sigma_e)$ | log $e$ persistence & std. dev | | $(0.91, 0.92)$ | | | | | $(0.91, 0.92)$ | |
| $\theta$ | Retention function curvature | | 0.181 | | | | | 0.181 | |
| $A^{illiq}/Z$ | Illiquid assets to post-tax inc. | | | | | | | | 4.83 |
| $\zeta(1+r)$ | Illiquid-liquid spread | | | | | | | | 0.08 |
| $\nu$ | Adjustment probability | | | | | | | | 0.089 |

*Note*: For analytical models (RA, TA, BU, TABU, ZL), the effective share of hand-to-mouth (HtM) is the parameter $\mu$. For quantitative models (HA-hi-liq, HA-one, HA-two), it is the share of income accruing to agents that have zero assets in their liquid accounts, and so an MPC of 1 out of transfers to that account.

Figure 1(b) shows the outcome of this exercise. The static MPC $M_{00}$ is too low: as Kaplan and Violante (2022) have argued, this model cannot explain the high MPCs in the data. The following iMPCs are too low as well. In fact, as figure 2(b) shows, all iMPCs are close to those of an RA model. Given that $\mathbf{M}$ is a sufficient statistic for the response to fiscal policy, this suggests that this model will behave similarly to RA in general equilibrium, reminiscent of the famous Krusell and Smith (1998) quasi-aggregation result.

An alternative way to approach calibration is to select $\beta$ to hit a target for $M_{00}$. We call this the *HA-one* model. As table 2 shows, this procedure delivers an annual $\beta = 0.87$: agents have to be quite impatient to allow themselves to stay close to the borrowing limit where their MPCs are high. Consequently, the average ratio of assets to after-tax income is only $A/Z = 0.21$, very low relative to the data. While this makes HA-one a challenging model for general equilibrium analysis, from the perspective of household behavior, this model turns out to have a much better fit to the iMPC data: as figure 2(a) shows, once we calibrate the model to hit $M_{00}$, it also fits the other $M_{t0}$ from the data very well. Figure 3(e) plots other columns of the $\mathbf{M}^{one}$ matrix.

Note, in fact, from figure 3, that the iMPC matrix of the HA-one model looks extremely similar to that of our calibrated TABU model. This is not a coincidence, as can be understood by studying the well-known *zero liquidity* limit of the one-account model. As we show in appendix D.4, if we keep recalibrating $\beta$ as we take the limit $A \to 0$, the resulting steady state is tractable and admits a closed form solution for its iMPC matrix, $\mathbf{M}^{ZL}$. Just like TABU, $\mathbf{M}^{ZL}$ is described by four parameters $r, \beta, \lambda, \mu$, and, in fact, has the same first column as $\mathbf{M}^{TABU}$. This explains the similarity between TABU and the HA-one model, which has low enough liquidity that it is similar to the ZL model.[27]

[27] In spite of their similarities, ZL and TABU are not exactly identical. The distinction is due to the fact that the

### 4.3 The two-account heterogeneous-agent model

We saw that the one-account HA model faces a tradeoff between hitting a reasonable ratio of assets to post-tax-income $A/Z$ (in HA-hi-liq) and hitting a reasonable $M_{00}$ (in HA-one). This is a well-known tradeoff in the literature. The leading model that is able to match both $M_{00}$ and a reasonable $A/Z$ is the two-account model (see Kaplan and Violante 2014, 2022).[28] Here we model illiquidity a la Calvo as in Bayer, Born and Luetticke (2024), because this is computationally tractable and more amenable to analytical results.[29] In the two-account model, or *HA-two*, households choose consumption $\widetilde{c}_{it}$, which gives them flow utility $u\left(\widetilde{c}_{it}\right)$. Otherwise, the objective is (26), the same as for the HA-one model, with CES utility $u$, which households maximize subject to the following budget and borrowing constraints:

$$
\begin{aligned}
\widetilde{c}_{it} + a_{it}^{liq} &= \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di} Z_t + (1+r)\,(1-\zeta)\, a_{it-1}^{liq} - d_{it} \cdot 1_{\{adj_{it}=1\}} && (27)\\
a_{it}^{illiq} &= (1+r) a_{it-1}^{illiq} + d_{it} \cdot 1_{\{adj_{it}=1\}} && (28)\\
a_{it}^{liq} &\geq 0, \quad a_{it}^{illiq} \geq 0 && (29)
\end{aligned}
$$

Here, $d_{it}$ represents net transfers from the liquid to the illiquid account. Agents are only able to transfer funds between accounts when $adj_{it} = 1$, which occurs iid with probability $\nu$. Both liquid and illiquid accounts are invested in bonds earning return $r$, but holding assets in the liquid account incurs a flow cost of $\zeta(1+r)a_{it-1}^{liq}$, proportional to the value of liquid assets entering period $t$. This cost is paid to a perfectly competitive financial intermediary providing liquidity services. Defining aggregate assets as $a_{it} \equiv a_{it}^{illiq} + a_{it}^{liq}$ and consolidating (27) and (28), we still have the constraint in (9) after defining consumption inclusive of the liquidity services from financial intermediation, $c_{it} \equiv \widetilde{c}_{it} + (1+r)\zeta a_{it-1}^{liq}$.

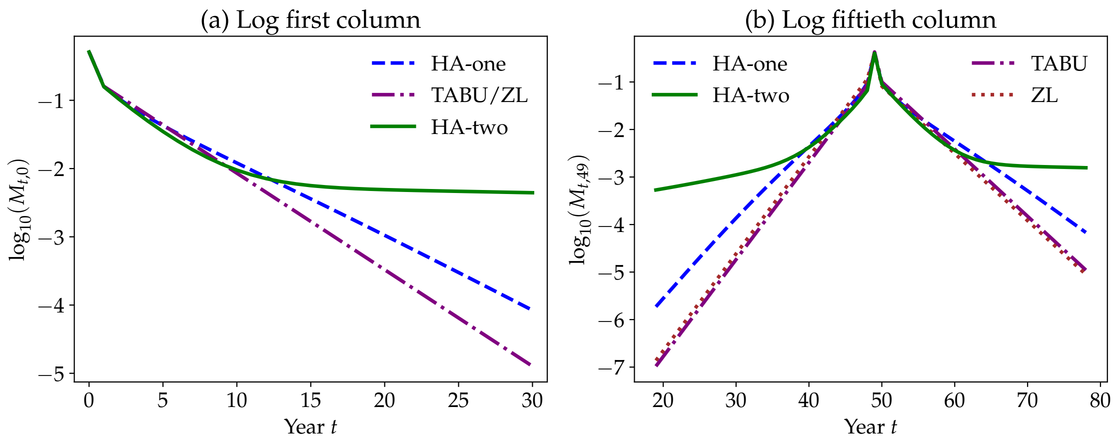
The first-order conditions, policy functions, and the stationary distribution of this model are described in appendix D.5. In the steady state, agents accumulate wealth in their liquid accounts to self-insure against fluctuations in their income. Self-insurance is costly as it incurs a return penalty of $\zeta\,(1+r)$ per period. Households therefore keep some of their wealth in their illiquid account, where it earns the higher rate of return $1+r$. If their income falls too much, they hope to be able to take distributions from this account. If the return penalty is sufficiently large, households

ZL model has a slightly stronger anticipated spending response, as derived and explained in appendix D.4. Another derivation of the **M** matrix for a zero-liquidity model with slightly different features is available in Bilbiie (2024), who also considers countercyclical income risk (see also Pfäuti and Seyrich 2022). To our knowledge, the analytical relationship with the TABU model is new.

[28] This model is often called the "two-asset" model in the literature. We prefer the name "two-account" for our model because households have multiple accounts they can invest in, but the financial intermediaries that ultimately provide these accounts invest them in any number of assets. For instance, in our current environment, they only invest in one asset, government bonds. In the quantitative environment of sections 6–7, both accounts will be invested in both bonds and firm equity—which we think is reasonable, given that in practice both bonds and stocks are held in liquid accounts (e.g. bank deposits and brokerage accounts) and illiquid accounts (e.g. pension funds).

[29] We conjecture that our findings are robust to an alternative formulation where illiquidity takes the form of a fixed cost of adjustment.

Figure 4: log iMPCs out of unexpected and expected income shocks




keep little wealth in their liquid account and most of their wealth in the illiquid account. This leaves them exposed to hitting the borrowing constraint after negative income shocks; thus, they are "wealthy hand-to-mouth" (Kaplan, Violante and Weidner 2014) and simultaneously have high MPCs and high wealth.

We calibrate this model by selecting $\beta$ and $\nu$ to meet targets for $M_{00}$ and aggregate assets $A/Z$. This calibration delivers a very reasonable $\beta = 0.93$, and an annual probability of accessing the illiquid account of $\nu = 0.089$. This is about half as large as the frequency in Kaplan and Violante (2014), which corresponds to $\nu = 0.168$, likely because our target for $M_{00}$ is somewhat higher.

Figure 2(a) shows that the iMPCs after an unanticipated shock (column $s = 0$) in the HA-two model fit the data well, as they do in the HA-one, ZL, and TABU models. However, among these four fitting models, only HA-two and TABU can be calibrated to large aggregate assets $A/Z$. Figure 3(f) plots multiple columns of the model's iMPC matrix. The stable long-run pattern is evident here as well.

Overall, figures 2 and 3 suggest that models with very different primitives, once calibrated to the existing evidence on iMPCs out of *unexpected* income shocks, predict similar tent-shaped iMPCs out of *expected* income shocks. Given the lack of good empirical evidence on these iMPCs, this is reassuring.

## 4.4 The long-term spending response in heterogeneous-agent models

We have found four models that can fit the existing evidence on the year 0 through year 5 spending behavior after unanticipated income shocks in figure 2(a): TABU, HA-one, ZL, and HA-two. While these models share very similarly-shaped **M** matrices, they also differ in subtle ways, which turn out to matter for the effects of fiscal policy. In this section, we examine these subtle differences.

Figure 4(a) plots the common logarithm of the first column of **M** out to 30 years, for each of these four models. HA-one, TABU, and ZL settle on exponentially decaying iMPCs $M_{t0}$ rather quickly. HA-two implies significantly greater spending further out, with iMPCs that eventually decay at a much slower rate. This is because, after a decade, most assets remaining from an income shock are in the illiquid account, and thereafter are depleted slowly. Figure 4(b) shows similar, slower decay rates in the right and left tails of the HA-two iMPCs after anticipated shocks (here for $s = 50$).[30]

There is currently no direct evidence that can tell apart the tail behavior shown in figure 4. Evidence from Fagereng et al. (2021) indicates that after year 5, most of any remaining increase in assets is in investments such as stocks, bonds, and mutual funds (see section 3.2). Since these investments may be fairly illiquid, and since estimated spending propensities out of illiquid wealth tend to be low (e.g. Di Maggio, Kermani and Majlesi 2020, Chodorow-Reich, Nenov and Simsek 2021), we conjecture that the HA-two model may provide a more accurate picture of tail spending behavior. However, more empirical research on long-term saving and spending responses to income shocks is needed before drawing definitive conclusions. The next section shows that this distinction matters for the quantitative efficacy of fiscal stimulus.

# 5 Fiscal policy according to the intertemporal Keynesian cross

We now solve the intertemporal Keynesian cross, obtaining the impulse response to a government spending shock when iMPCs are generated by the models in section 4. As it turns out, the results depend crucially on the financing of fiscal policy. We first consider the case of balanced-budget policy, and then move to the general case with deficit-financed spending.

For simplicity, we will focus on the canonical RA and TA models, which do not match iMPCs, and the HA-one and HA-two models, which do. To better understand fiscal policy transmission in the two HA models, we will also study the TABU model, which has a similar fit to iMPCs but allows for analytical results. The proofs for this section are available in appendices E.1–E.3; for completeness, we cover fiscal policy in the other models of section 4 in appendix E.4.

It is standard in the literature to summarize the effects of government spending on output using a "multiplier". We study both the *impact multiplier* $dY_0/dG_0$ and the *cumulative multiplier* $\sum_{t=0}^{\infty}(1+r)^{-t}dY_t / \sum_{t=0}^{\infty}(1+r)^{-t}dG_t$ (see Mountford and Uhlig 2009 and Ramey 2019). The latter is sometimes considered a more useful measure of the overall impact of policy, capturing propagation as well as amplification of fiscal shocks.[31]

## 5.1 Balanced-budget fiscal policy

Our first result is a sharp characterization of the effects of balanced-budget fiscal policy.

[30] This relationship between unanticipated and anticipated shocks can be derived as a consequence of the result in appendix D.1. While figure 4 shows that the TABU model cannot match all the iMPCs of HA-two, we conjecture that adding a small fraction of another BU household, with $\lambda$ close to 1, might achieve a good fit. If this household holds a large share of assets, this modification would also improve the fit to capital gains discussed in section 6.3. See Auclert et al. (2023c) for a model that mixes different types of BU households.

[31] The literature also sometimes refers to intermediate objects such as $\sum_{t=0}^{T}(1+r)^{-t}dY_t / \sum_{t=0}^{T}(1+r)^{-t}dG_t$ for some $T > 0$. This number is generally in between our reported impact and cumulative multipliers.

**Proposition 3** (Balanced-budget policy)**.** *Assume a unique equilibrium, and that the fiscal policy* $\{d\mathbf{G}, d\mathbf{T}\}$ *has a balanced budget, that is,* $d\mathbf{G} = d\mathbf{T}$*. Then, the fiscal multiplier is 1 at every date,* $d\mathbf{Y} = d\mathbf{G}$*.*

This result can easily be shown by guessing and verifying that $d\mathbf{Y} = d\mathbf{G} = d\mathbf{T}$ satisfies (13). Hence, if there is a unique solution for $d\mathbf{G} = d\mathbf{T}$, then $d\mathbf{Y} = d\mathbf{G}$ is this solution. The intuition is simple: output and therefore pre-tax income rise by exactly the shock to spending, and then taxes rise by the same amount. This leaves after-tax income, and therefore consumption, unchanged.[32]

As long as fiscal policy keeps the budget balanced, proposition 3 implies that the fiscal multiplier in every period is exactly equal to 1, irrespective of iMPCs $\mathbf{M}$. For instance, HA and RA economies, despite very different iMPCs, have the same balanced-budget multiplier.

It is critical for this result that income and taxes have the same incidence across households. This ensures that when aggregate income and taxes increase by the same amount, individual agents' after-tax incomes are unchanged. If, alternatively, income and taxes have different incidence, there is an additional redistribution effect between income-earners and taxpayers that changes the multiplier. For instance, appendix E.1 shows that when taxes are raised lump-sum at the margin, the fiscal multiplier is less than 1, because then taxpayers have higher short-term iMPCs than income-earners.

### 5.2 Deficit-financed fiscal policy

While iMPCs are irrelevant for balanced-budget policies, they are central with deficit financing.

**Proposition 4** (Deficit-financed policies)**.** *Assume a unique equilibrium. The output response to a fiscal policy shock* $\{d\mathbf{G}, d\mathbf{T}\}$ *is the sum of the government spending policy* $d\mathbf{G}$ *and the effect on consumption* $d\mathbf{C}$*,*

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}}. \tag{30}$$

*The consumption response* $d\mathbf{C}$ *only depends on the path of primary deficits* $d\mathbf{G} - d\mathbf{T}$*. In particular, holding the deficit fixed, government spending has a greater effect on output than transfers do.*

Proposition 4 shows that for non-balanced-budget policies, the consumption response is entirely driven by the *interaction* between iMPCs—which determine $\mathcal{M}\mathbf{M}$—and primary deficits $d\mathbf{G} - d\mathbf{T}$.

One implication of (30) is a clear relationship between government *spending* and *transfer* multipliers: assuming the same plan for deficit financing, the spending multiplier should equal the transfer multiplier plus one. This is not always found in empirical work: for instance, Ramey (2019) argues that transfer multipliers as in Romer and Romer (2010) tend to be larger than spending multipliers minus one. Our result suggests that this difference should be traceable to either different deficit financing for the two kinds of shocks, or some other difference that does not appear in (30), such as different monetary responses or different incidence of taxes.

[32] Gelting (1941) and Haavelmo (1945) were the first to spell out this logic in the context of a static IS-LM model.

In the remainder of this section, we explore the interaction between iMPCs and primary deficits in (30): first for analytical models, and then for heterogenous-agent models.

### 5.3 Multipliers in analytical models

We begin with the RA and TA models. These models do feature multiple equilibria, since for them, $\mathbf{M1} = \mathbf{1}$: adding a constant to output at every date continues to satisfy the IKC (13). However, if $\lim_{t\to 0} dG_t = \lim_{t\to 0} dT_t = 0$, there exists a unique equilibrium $d\mathbf{Y}$ satisfying $\lim_{t\to 0} dY_t = 0$. We restrict our analysis to these equilibria in the propositions below.[33]

Woodford (2011) and Bilbiie (2011) prove the following result for the RA model:

**Proposition 5** (Fiscal policy in the RA model)**.** *In the RA model, $d\mathbf{Y} = d\mathbf{G}$ irrespective of $d\mathbf{T}$. In particular, impact and cumulative multipliers are equal to 1.*

This follows from (30), where the intertemporal government budget $\mathbf{q}'(d\mathbf{G} - d\mathbf{T}) = 0$ implies that $\mathbf{M}\,(d\mathbf{G} - d\mathbf{T}) = (1-\beta)\,\mathbf{1q}'\,(d\mathbf{G} - d\mathbf{T}) = 0$, and therefore that the second term is zero. A simple interpretation is that Ricardian equivalence holds in the RA model, so any policy is equivalent to a balanced-budget policy (proposition 3) and has a unit multiplier.

The solution (30) is also tractable in the TA model, which has been very influential for the study of fiscal policy, largely because it can be solved with pen and paper and offers insightful results (see Bilbiie and Straub 2004, Galí et al. 2007). We prove the following:

**Proposition 6** (Fiscal policy in the TA model)**.** *In the TA model, $d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}(d\mathbf{G} - d\mathbf{T})$. The impact multiplier is equal to $\frac{1}{1-\mu} - \frac{\mu}{1-\mu}\frac{dT_0}{dG_0}$, but the cumulative multiplier is 1.*

The TA model is no longer Ricardian and therefore generally produces non-unitary multipliers when $d\mathbf{G} \neq d\mathbf{T}$. However, the model's non-Ricardian behavior is entirely driven by its fraction $\mu$ of hand-to-mouth agents, which spend their income entirely in the same period as it is received. This static relationship between income and consumption leads output to follow the static Keynesian cross (5), with $\mu$ playing the role of $mpc$.[34] The outcome is an impact multiplier of $\frac{1}{1-\mu}$ for spending that is entirely deficit-financed ($dT_0 = 0$), and a transfer multiplier of $\frac{\mu}{1-\mu}$. Interestingly, however, the model *still* generates unitary cumulative multipliers, since consumption declines as soon as deficits are turned into surpluses.[35]

Neither RA nor TA can produce iMPCs consistent with the data. We next study a model that can, the TABU model, and later show that the insights we gather here carry over numerically to our heterogeneous-agent models HA-one and HA-two.[36]

[33] This selection is standard in the literature with a constant real interest rate rule (see e.g Woodford 2011). One justification could be that, at a distant point in the future, we revert to a Taylor rule that ensures determinacy. Another is that this selection is the unique limiting equilibrium with a rule $i_t = r + \phi\pi_{t+1}$ as $\phi$ approaches 1 from above.

[34] Note that $\mu$ is slightly less than the actual MPC out of current income in the model, which includes the consumption response of the permanent-income agents. These agents do not appear in the multiplier because they are Ricardian, and the present value of their after-tax income is unchanged.

[35] This unitary cumulative multiplier with constant $r$ was earlier noted by Bilbiie, Monacelli and Perotti (2013).

[36] See appendix E.3 for a similar formula giving fiscal multipliers in the ZL model and an analog to corollary 1. The BU model is the special case of proposition 7 where $\mu = 0$.

**Proposition 7** (Fiscal policy in the TABU model)**.** *Consider a TABU model with parameters $\lambda$, $\mu$, $\beta$ and $r$, and assume that $\beta(1+r) < 1$. Then, there exists a unique solution $\{dY_t\}$ to the IKC (13) for given fiscal policy $\{dG_t, dT_t\}$ generating a path of debt $dB_t = \sum_{s \le t}(1+r)^s(dG_s - dT_s)$, and it is given by:*

$$dY_t = dG_t + \frac{\mu}{1-\mu}(dG_t - dT_t) + (1+r)\frac{1-\frac{\lambda}{1+r}}{1-\mu}\left(\frac{1}{\lambda} - \beta(1+r)\right)\sum_{s=0}^{\infty}(\beta(1+r))^s dB_{t+s} \tag{31}$$

The TABU output response (31) begins with the same two terms as the TA response in proposition 6, reflecting static Keynesian cross forces. But there is a new, third term, which depends on current and future government debt. This term arises because the bond-in-utility households have elevated iMPCs out of recent past and future income. When they receive unusually high income, these households accumulate asset balances in excess of their steady-state target, which they later spend down, and anticipation of future income from this spending triggers even more spending beforehand. This is the dynamic income-spending feedback from the intertemporal Keynesian cross. The TA model, by contrast, lacks this term because its households are at one of two extremes—either completely Ricardian, or completely hand to mouth—and no one tries to spend down excess assets when the government incurred a deficit in the past.

One useful way to see the difference between the TABU and TA models—and the role of intertemporal MPCs—is the following. Given a fixed target for $M_{00}$, the TABU model can match it in two ways: either with many hand-to-mouth agents as in the TA model (high $\mu$), or with BU households that quickly spend down excess assets (low $\lambda$). In the second case, iMPCs immediately off the main diagonal, such as $M_{10}$, are higher, and one can show that the coefficient on future debt in (31) is also higher, leading to a larger cumulative multiplier in the presence of debt.

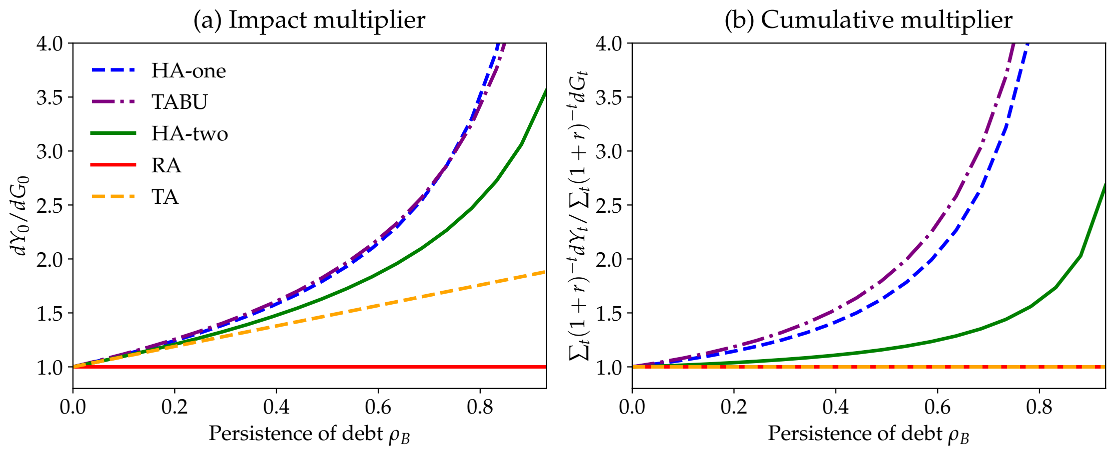
**Corollary 1.** *Consider a TABU model with $\beta(1+r) < 1$. Recalibrating $\mu$ and $\lambda$ to match a higher $M_{10}$ with the same $M_{00}$ increases the cumulative multiplier whenever $d\mathbf{B} \ge 0$ and $dB_t > 0$ for some $t$.*

Proposition 7 and corollary 1 are important because they show that, for deficit-financed policy, intertemporal MPCs matter: they interact with the level of debt in a way that is not summarized by the contemporaneous MPC $M_{00}$. Larger consumption responses to recent past and future income imply a larger cumulative multiplier.[37]

The TABU model is a convenient vehicle to make these points analytically. We show next that the same insights carry over numerically in our heterogeneous-agent models.

[37] Note that it is not just the existence of positive iMPCs that matters here—since households eventually spend the full present value of their income in any model—but the fact that these households have a frontloaded consumption response. In parallel work, Angeletos et al. (2023) prove a related result: when $M_{10}$ increases in a TABU model, "self-financing" is more likely under a fiscal rule that holds the tax rate fixed. A limitation of Corollary 1 is that, when permanent-income agents are added to the model, the cumulative multiplier goes back to 1. Angeletos et al. (2023) explain why this is not an empirically relevant result.

Figure 5: Multipliers according to the IKC




*Note.* These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary $\rho_B$ in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

### 5.4 Multipliers in heterogeneous-agent models and quantitative comparison

To study fiscal policy in heterogeneous-agent models, we numerically compute the output response to a specific fiscal policy shock. We assume that government spending declines exponentially at rate $\rho_G$, $dG_t = \rho_G^t dG_0$. Taxes are chosen such that public debt is given by $dB_t = \rho_B(dB_{t-1} + dG_t)$. In this formulation, $\rho_B$ is the degree of deficit financing: if $\rho_B = 0$, the policy keeps a balanced budget, while for greater $\rho_B$, the policy leads to a greater deficit. We compute the responses to this shock for various degrees of deficit financing and for the main models considered in figure 2, and report the corresponding impact and cumulative multipliers.

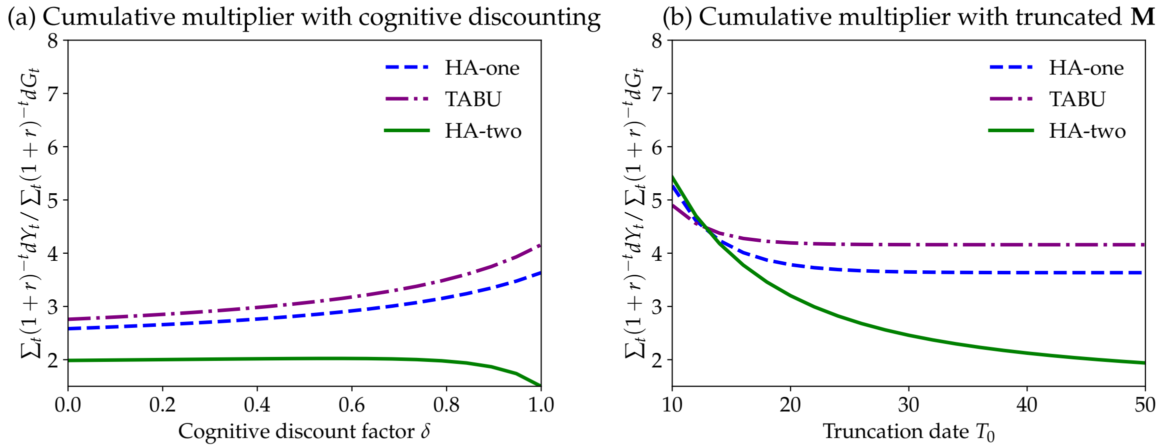
Figure 5 displays these multipliers. As in proposition 3, both impact (left panel) and cumulative multipliers (right panel) are exactly equal to 1 when fiscal policy balances the budget, irrespective of iMPCs. As the degree of deficit financing $\rho_B$ rises, however, the models separate.

Impact multipliers increase with $\rho_B$ in all models except RA, where multipliers remain at 1, as in proposition 5. This emphasizes that high impact multipliers can be generated entirely in models with high static MPC $M_{00}$. Cumulative multipliers, on the other hand, crucially depend on intertemporal MPCs, such as $M_{10}$.

Despite having high $M_{00}$, the TA model predicts cumulative multipliers of one, independent of deficit financing, as in proposition 6. The two HA models, as well as TABU, by contrast, find cumulative multipliers that increase with deficit financing, confirming the insights in proposition 7 and corollary 1.[38]

[38]These results are not special to our assumption of small shocks starting from the steady state. In appendix E.5, we demonstrate that the fiscal multipliers exhibit limited nonlinearity and state dependence in the models we consider.

Figure 6: Effect of adding cognitive discounting to, and of truncating the tails of **M**




*Note.* In this calibration, $\rho_G = \rho_B = 0.76$. See section 7.1 for details on calibration choices.

### 5.5 Differences between TABU, HA-one, and HA-two

In figure 5, although the multipliers for TABU, HA-one, and HA-two are qualitatively similar, there are noticeable quantitative differences. This is for two reasons. First, these models have different anticipatory effects, which are not directly pinned down by the data. Second, these models have different long-term iMPCs, as documented in section 4.4.

Figure 6(a) explores the role of anticipatory effects, by allowing for *cognitive discounting* as in Gabaix (2020) to dampen these effects, with $\delta = 0$ corresponding to no anticipation and $\delta = 1$ being the original model (see appendix E.6 for details). We see that as $\delta \to 0$, the multipliers become more similar across models.[39]

Figure 6(b) explores the role of long-term iMPCs more generally, by setting iMPCs to zero past a certain distance $T_0$ from the diagonal ("truncating" **M**), and scaling up the remaining iMPCs to enforce budget balance. This raises multipliers across the board, since it pushes up both $M_{00}$ and $M_{10}$. It has an especially strong effect, however, for HA-two, because it reallocates HA-two's higher long-term iMPCs (section 4.4) toward the main diagonal. For low enough $T_0$, multipliers converge to similar levels. This indicates that HA-two's higher long-term iMPCs are responsible for its lower multipliers: they make the model slightly more Ricardian, weakening the short-term income-consumption feedback in the intertemporal Keynesian cross.

### 5.6 Taking stock: the interaction of iMPCs and deficits

This section established that it is the *interaction* of the shape of iMPCs **M** and primary deficits that drives multipliers. By matching $M_{00}$, the TA model generates high impact multipliers, but

[39]Two forces shape whether output $d\mathbf{Y} = \mathcal{M}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$ increases or decreases as $\delta \to 0$: it increases because future taxes are less anticipated; it decreases because the multiplier $\mathcal{M}$ is lower. Figure E.5 separates these two effects. Anticipation of taxes matters more for the two-account model because its households care more about income in the relatively distant future, as seen in section 4.4.

not high cumulative multipliers. By matching $M_{00}$ as well as other, "off-diagonal" elements such as $M_{10}$, both TABU and heterogeneous-agent models generate high impact *and* high cumulative multipliers. The exact quantitative magnitude of the multipliers depends not only on the currently observable elements of $\mathbf{M}$ summarized in figure 1, but also on anticipation effects and on the longer-term spending response to income, for which more empirical research is needed. In the next two sections, we show that these conclusions continue to hold in more quantitative environments provided that the models match one additional set of intertemporal MPCs—those out of capital gains.

## 6 Allowing for interest rate effects and capital gains

Sections 2–5 showed that, under some restrictions on the environment, the general equilibrium effects of fiscal policy can be reduced to a single equation, the intertemporal Keynesian cross, with the iMPC matrix $\mathbf{M}$ capturing all relevant details of the model.

We now take a step toward relaxing these restrictions. In this section, we characterize household behavior when real interest rates are no longer constant and assets are subject to valuation effects. Our main result is that, surprisingly, under two common assumptions on preferences and portfolio shares, only a single additional sequence, $\mathbf{m}^{cap}$—the consumption response to unanticipated capital gains—is needed to capture household behavior together with $\mathbf{M}$. Comparing $\mathbf{m}^{cap}$ across models with evidence on the MPC out of capital gains, we argue that the evidence strongly favors the two-account model.

### 6.1 Extending the consumption function

We now extend the environment of section 2.2. We allow for a time-varying real rate $r_t$, and for a second asset besides government debt—firm equity. There is a unit mass of shares, each of which trades at an end-of-period price $p_t$ and pays a dividend $d_t$ at time $t$. As we explain in the next section, the value of these shares derives from firms' capital, their market power, and fluctuations in profits from sticky prices.

Households can frictionlessly exchange bonds and shares, so that a no arbitrage condition equates their expected returns at all $t \geq 0$,

$$1 + r_t = \frac{p_{t+1} + d_{t+1}}{p_t} \tag{32}$$

This condition does not pin down the realized return on shares at date 0, $\frac{p_0+d_0}{p_{-1}}$, which can deviate from the steady state expected return $r_{ss}$ due to unexpected capital gains and losses resulting from changes in the present discounted value of $d_t$.[40]

[40] Repeatedly applying (32), we see that $p_0 + d_0 = \sum_{t\geq 0} \left( \prod_{s<t} \frac{1}{(1+r_s)} \right) d_t$.

Since steady-state expected returns are equal across the two assets, steady state portfolio shares of households are indeterminate. For all models except HA-two, we denote the equity portfolio share of household $i$ entering period 0 by $\varpi_{i,-1}$. For HA-two, each household $i$ has two equity shares, one in the liquid and the other in the illiquid account, $\varpi_{i,-1}^{liq}$ and $\varpi_{i,-1}^{illiq}$. The equity shares satisfy the initial condition $\int \varpi_{i,-1} a_{i,-1} di = p_{-1}$ in all models except HA-two, and $\int \varpi_{i,-1}^{illiq} a_{i,-1}^{illiq} di + \int \varpi_{i,-1}^{liq} a_{i,-1}^{liq} di = p_{-1}$ in HA-two.

In this environment, appendix F.1 shows that the consumption function becomes:

$$C_t = \mathcal{C}_t\left(\{Z_s, r_s\}, p_0 + d_0\right) \tag{33}$$

As before, $Z_t = w_t N_t - T_t$ is aggregate after-tax labor income, now including a possibly time-varying real wage $w_t$; and $p_0 + d_0$ is the cum-dividend value of shares. Linearizing this equation around the steady state, we obtain:

$$d\mathbf{C} = \mathbf{M} \cdot d\mathbf{Z} + \mathbf{M}^r \cdot d\mathbf{r} + \mathbf{m}^{cap}\, d\text{cap}_0 \tag{34}$$

Equation (34) shows that in this broader environment, two new terms emerge as determinants of consumption behavior, beyond $\mathbf{M} \cdot d\mathbf{Z}$. The first new term, $\mathbf{M}^r \cdot d\mathbf{r}$, captures the consumption response to changes in real interest rates $d\mathbf{r}$; here, $\mathbf{M}^r \equiv \left[\frac{\partial \mathcal{C}_t}{\partial \log(1+r_s)}\right]_{ts}$ is the Jacobian of consumption to interest rates. The second new term, $\mathbf{m}^{cap}\, d\text{cap}_0$, captures the consumption response to the the surprise capital gain $d\text{cap}_0 \equiv d\left(p_0 + d_0\right)$ at date 0; here, $m_t^{cap} \equiv \frac{\partial \mathcal{C}_t}{\partial (p_0 + d_0)}$ is the impulse response of consumption to an unexpected capital gain on the unit mass of shares.

### 6.2 iMPCs as sufficient statistics for interest rate effects

We have identified three sufficient statistics for household behavior: $\mathbf{M}$, $\mathbf{M}^r$, and $\mathbf{m}^{cap}$. We argued in sections 3–5 that data could meaningfully discipline $\mathbf{M}$, and in the next section we will discuss how similar data can also be used to discipline $\mathbf{m}^{cap}$—which is ultimately a set of intertemporal MPCs, too, just out of capital gains. In principle, we could also seek direct data on $\mathbf{M}^r$. Here, we provide an alternative route: we show that, for a broad class of models, $\mathbf{M}^r$ is directly determined by $\mathbf{M}$ and $\mathbf{m}^{cap}$.

**Proposition 8.** *For the RA, TA, HA-one, ZL, and HA-two models, assuming an elasticity of intertemporal substitution of $\sigma = 1$, and equal initial equity portfolio shares $\varpi_{i,-1} = \varpi$ (with $\varpi_{i,-1}^{liq} = \varpi_{i,-1}^{illiq} = \varpi$ in HA-two) for all $i$, we have:*

$$\mathbf{M}^r = -C\left(\mathbf{I} - \left(1 - \frac{rA}{C}\right)\mathbf{M}\right)\mathbf{U} + (1+r)A\,\mathbf{m}^{cap}\mathbf{1}' \tag{35}$$

*where $\mathbf{U}$ is an upper-triangular matrix of ones, $A = \int a_i di$ and $C = \int c_i di$ are aggregate assets and consumption in the steady state.*

Proposition 8, proved in appendix F.2, is a nontrivial result. It states that across all models listed, the entire matrix $\mathbf{M}^r$ of date-$t$ consumption responses to date-$s$ interest rate changes can be expressed using iMPCs $\mathbf{M}$ and the consumption response to capital gains $\mathbf{m}^{cap}$—neither of which have any obvious connection to interest rate changes.

For intuition, suppose first that assets are zero, $A = 0$. Then, (35) simply reads $\mathbf{M}^r = -C\,(\mathbf{I} - \mathbf{M})\,\mathbf{U}$.[41] Here, higher iMPCs $\mathbf{M}$ out of labor income lower the sensitivity of consumption to interest rates, as they imply a shorter effective planning horizon for consumption.[42] When assets are positive, $A > 0$, higher iMPCs $\mathbf{m}^{cap}$ out of capital gains also imply a shorter effective planning horizon, and similarly lower the sensitivity of consumption to interest rates.

As stated, the result in proposition 8 requires equal equity shares across all households. In heterogeneous-agent models with a single account, this is a common baseline assumption. For the two-account model, proposition 8 also requires equal equity shares across the liquid and illiquid accounts, which is a less common assumption. In both cases, however, the assumption only matters quantitatively to the extent that it affects $\mathbf{m}^{cap}$, which in the next section we argue is low regardless.[43] We also show in appendix F.5 that a model in which all equity is held in illiquid accounts is quantitatively close to the equal-equity-share model considered here.

The assumption that the elasticity of intertemporal substitution equals 1 is relaxed in appendix F.3, although this requires generalizing $\mathbf{m}^{cap}$ to a richer object $\mathbf{M}^{cap}$ that captures the iMPCs out of capital gains in arbitrary periods, not just date 0. Also using $\mathbf{M}^{cap}$, the proposition can be extended to the BU and TABU models (appendix F.4).[44]

### 6.3 iMPCs out of capital gains in models and data

The previous section showed that there is a tight link between iMPCs out of income and capital gains on the one hand, and the consumption response $\mathbf{M}^r$ to interest rates on the other. This makes it important to understand iMPCs out of capital gains, both because of their direct role in (34), and because of their indirect role via $\mathbf{M}^r$.

Figure 7(a) plots $\mathbf{m}^{cap}$, the response of consumption to a one-time date-0 capital gain across models.[45] RA, TA, and HA-two all have relatively low $\mathbf{m}^{cap}$. For the RA model, this is because it has low iMPCs in general (figure 7(b)). For TA, it is because assets are only held by unconstrained households. For HA-two, it is because most assets are held in the illiquid account, and iMPCs out of the illiquid account are small. iMPCs are even somewhat smaller if all equity is held in illiquid accounts (appendix F.5). By contrast, TABU has very high $\mathbf{m}^{cap}$. This is because even the unconstrained agents in TABU, which own all assets, have sufficiently high MPCs to push the

[41] This result can be combined with section 6.1 to show that, when $A = B = 0$, the effect of monetary policy $d\mathbf{r}$ is simply $d\mathbf{Y} = -C\mathbf{U}d\mathbf{r}$ , i.e. the representative-agent outcome. This is an instance of the Werning (2015) neutrality result.

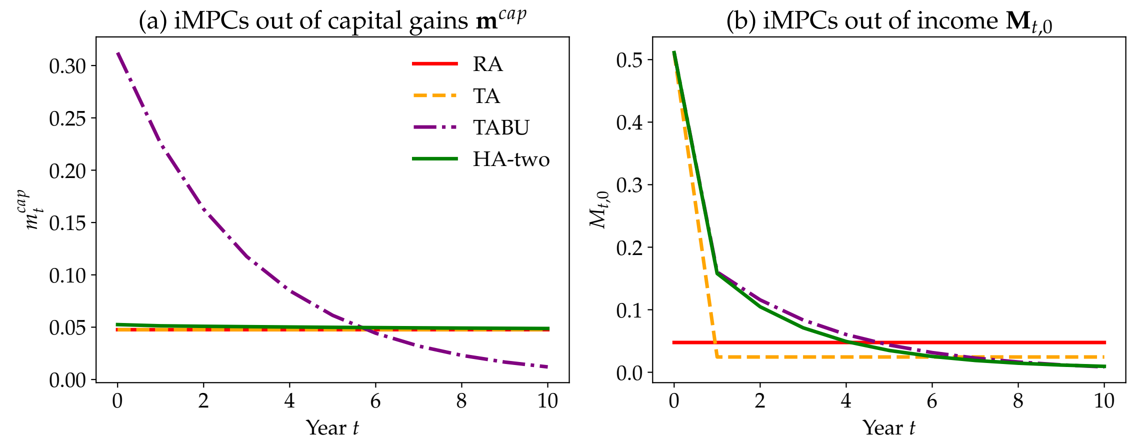
[42] See also Auclert (2019), Auclert, Rognlie, Souchier and Straub (2021b), Farhi, Olivi and Werning (2022), as well as Koby and Wolf (2020) for a related relation between the elasticities of investment to interest rates vs output.

[43] In practice, equity does appear in both liquid accounts (e.g. easy-to-trade brokerage accounts) and illiquid accounts (e.g. pensions), but the equity share may not be the same across both. The $\mathbf{m}^{cap}$ that is relevant for proposition 8 is the response to a uniform capital gain across all accounts.

[44] Wolf (2023a) has previously shown a relationship between $\mathbf{M}$ and $\mathbf{M}^r$ in analytical models.

[45] From now on, we drop HA-one, as it cannot be calibrated to a realistic target for total assets (see table 2).

Figure 7: iMPCs out of capital gains vs income




impact MPC $m_0^{cap}$ above 0.30.

In the data, the MPC out of capital gains is generally estimated to be low, consistent with the first set of models. For instance, Di Maggio et al. (2020) estimate an $m_0^{cap}$ of 5 cents, and Chodorow-Reich, Nenov and Simsek (2021) estimate a relatively flat path of $m_t^{cap}$ around 2.8 cents. Among the two models that can match both $\mathbf{M}$ and aggregate assets, TABU and HA-two, only HA-two is consistent with this evidence. This provides further support for HA-two being a suitable household model for quantitative fiscal policy analysis, even in the presence of interest rate and capital gains effects.

# 7 Fiscal policy in a quantitative HANK model

We now embed our consumption-savings models in a fully-specified quantitative general equilibrium environment, which relaxes several restrictive assumptions of the IKC environment in section 2. The main features of our quantitative environment are: (a) a realistic supply side, with output produced from capital and labor; (b) investment as a component of aggregate demand; (c) wage and price rigidities; and (d) a broader set of monetary policy rules, including active Taylor rules and a possible zero lower bound (ZLB).

It is well understood from the existing literature on fiscal multipliers that, with an active Taylor rule, real interest rates rise in response to expansionary fiscal policy. This tends to crowd out consumption and investment, especially when taxation is distortionary (eg, Woodford 2011, Drautzburg and Uhlig 2015). It is also is well understood that, at the zero lower bound (ZLB), these effects are reversed, with falling real interest rates tending to crowd in consumption and investment (eg, Christiano et al. 2011). This section shows that, while heterogenous-agent models also feature these standard effects, the multipliers they generate are nevertheless quite different, especially when spending is deficit-financed and the interaction effect in proposition 4 has bite.

### 7.1 Extended model setup

Here we describe the problem solved by firms and households; monetary and fiscal policy; and our equilibrium concept. Without loss of generality given our first-order perturbation solution, we assume that all agents have perfect foresight with respect to aggregates (see footnote 5).

**Production, sticky prices, and sticky wages.** Away from a real interest rate rule, inflation matters for the determination of aggregate demand. We adopt the microfoundations for nominal wage rigidities described in appendix A.3, with Rotemberg (1982)-like adjustment costs. With the disutility of labor specified as $v(N) = \gamma N^{1+\frac{1}{\phi}}$, this leads to a nonlinear wage Phillips curve:

$$\pi_t^w (1 + \pi_t^w) = \kappa^w \left( \mu^w \frac{\gamma N_t^{\frac{1}{\phi}}}{(C_t^*)^{-\sigma} (1-\theta) Z_t / N_t} - 1 \right) + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w) \tag{36}$$

describing the dynamics of wage inflation $\pi_t^w$ as a function of aggregate hours $N_t$, aggregate post-tax income $Z_t$, and a "virtual" consumption aggregate $C_t^* \equiv \left( \int_i \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di} c_{it}^{-\sigma} di \right)^{-\frac{1}{\sigma}}$ which summarizes how the distribution of consumption across the population affects the aggregate wealth effect on labor supply.

To accommodate production from capital and sticky prices, we now assume a standard two-tier production structure. Final goods firms aggregate intermediate goods with a constant elasticity of substitution $\mu^p / (\mu^p - 1) > 1$. Intermediate goods are produced by a mass one of identical monopolistically competitive firms, whose shares are traded, with price $p_t$ and dividends $d_t$ at time $t$, and owned by households. All firms have the same production technology, now assumed to be Cobb-Douglas in labor and capital, $y_t = F(k_{t-1}, n_t) = \Theta k_{t-1}^{\alpha} n_t^{1-\alpha}$, where $\Theta$ is a constant TFP term. Capital is subject to quadratic capital adjustment costs, so that investment $i_t \equiv k_t - (1-\delta) k_{t-1}$ to attain $k_t$ from $k_{t-1}$ requires an additional adjustment cost $\varphi\left(\frac{k_t}{k_{t-1}}\right) k_{t-1}$, where $\varphi(x) \equiv \frac{1}{2\delta\epsilon_I}(x-1)^2$, $\delta$ denotes depreciation, and $\epsilon_I$ is the sensitivity of gross investment to Tobin's $Q$. Finally, any firm chooses a price $\mathcal{P}_t$ in period $t$ subject to Rotemberg (1982) adjustment costs $\xi(\mathcal{P}_t, \mathcal{P}_{t-1}) \equiv \frac{1}{2\kappa^p(\mu^p - 1)} \left( \frac{\mathcal{P}_t - \mathcal{P}_{t-1}}{\mathcal{P}_{t-1}} \right)^2$, where $\kappa^p$ is the slope of the price Phillips curve.[46]

In this setting, an intermediate goods firm entering period $t$ with capital $k_{t-1}$ chooses its price $\mathcal{P}_t$, labor $n_t$ and capital for next period $k_t$ to maximize its value:

$$J_t(k_{t-1}) = \max_{\mathcal{P}_t, k_t, n_t} \left\{ \frac{\mathcal{P}_t}{P_t} F(k_{t-1}, n_t) - \frac{W_t}{P_t} n_t - i_t - \varphi\left(\frac{k_t}{k_{t-1}}\right) k_{t-1} - \xi(\mathcal{P}_t, \mathcal{P}_{t-1}) Y_t + \frac{1}{1+r_t} J_{t+1}(k_t) \right\} \tag{37}$$

subject to the requirement that it satisfies final goods firms' demand in each period at its chosen

[46] Since we solve the model to first order in aggregates, Rotemberg adjustment costs are equivalent to price setting à la Calvo (1983).

price:

$$F(k_{t-1}, n_t) = \left(\frac{\mathcal{P}_t}{P_t}\right)^{-\mu^p/(\mu^p-1)} Y_t \tag{38}$$

Since all intermediate goods firms are identical, in equilibrium they make the same choices $k_t = K_t$, $n_t = N_t$, and $\mathcal{P}_t = P_t$; moreover the stock price satisfies $p_t = \frac{1}{1+r_t}(d_{t+1} + p_{t+1})$, with the aggregate dividend equal to $d_t \equiv F(K_{t-1}, N_t) - \frac{W_t}{P_t} N_t - I_t - \varphi\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} - \xi(P_t, P_{t-1}) Y_t$.

As we show in appendix G.1, this setup generates a nonlinear Phillips curve for price inflation,

$$\pi_t (1 + \pi_t) = \kappa^p (\mu^p \cdot mc_t - 1) + \frac{1}{1+r_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (1 + \pi_t) \tag{39}$$

where $mc_t \equiv \frac{W_t/P_t}{F_{n,t}}$ is real marginal cost, as well as a set of two standard $Q$ theory equations for capital demand and the dynamics of investment.

**Households.** We consider the six structural models of consumption and savings of section 4 that can be calibrated to a realistic level of aggregate assets: RA, TA, BU, TABU, HA-hi-liq and HA-two (see table 2). We calibrate these models to $\sigma = 1$ and equal initial portfolio shares in bonds and equities for all households and all accounts. As shown in section 6, this means that in aggregate, these household models are entirely summarized by their $\mathbf{M}$ and their $\mathbf{m}^{cap}$.

**Monetary and fiscal policy.** The monetary authority now follows a Taylor rule:

$$i_t = r + \phi_\pi \pi_t \tag{40}$$

where the coefficient on inflation $\phi_\pi$ ensures determinacy[47] and $r$ is the steady-state interest rate. As in section 5.4, the government follows an AR(1)-type spending policy, $dG_t = \rho_G^t dG_0$, with $\rho_G \in (0, 1)$ and $dG_0$ equal to 1% of steady-state output. Taxes are chosen such that the path of public debt is given by $dB_t = \rho_B(dB_{t-1} + dG_t)$.

**Equilibrium.** Given initial values for the nominal wage $W_{-1}$, price level $P_{-1}$, government debt $B_{-1}$, and capital $K_{-1}$, and an initial distribution of households $i$ over their state variables such that the economy is initially at a steady state, and given exogenous sequences for fiscal policy $\{G_t, T_t\}$ that satisfy the government's intertemporal budget constraint, a (perfect-foresight) *general equilibrium* is a path for prices $\{P_t, W_t, \pi_t, \pi_t^w, r_t, i_t, p_t\}$ and aggregates $\{Y_t, K_t, N_t, C_t, d_t, B_t, G_t, T_t\}$, such that households optimize, unions optimize, firms optimize, monetary and fiscal policy follow their rules, and all markets clear: the goods market,

$$G_t + \int \widetilde{c}_{it} di + \zeta_t + I_t + \varphi\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} + \xi(P_t, P_{t-1}) Y_t \quad = \quad Y_t$$

[47] By usual arguments, RA and TA are determinate for $\phi_\pi > 1$. The winding number test (Auclert et al. 2023b) shows that the determinacy threshold for HA-hi-liq is 1.02. Determinacy thresholds for HA-two, BU and TABU are 1.05, 1.12 and 1.09, respectively. Our calibration to $\phi_\pi = 1.5$ therefore ensures that all of these models are determinate.

Table 3: Calibration of the quantitative environment

| Parameter | Description | Value | Parameter | Description | Value |
|---|---|---|---|---|---|
| $\alpha$ | Capital share | 0.294 | $\phi$ | Frisch elasticity of labor supply | 1 |
| $G/Y$ | Gov. spending-to-GDP | 0.20 | $\varepsilon_I$ | Investment elasticity to $q$ | 4 |
| $B/Y$ | Debt-to-GDP | 0.70 | $\kappa^p$ | Price flexibility | 0.23 |
| $K/Y$ | Capital-to-GDP | 2.26 | $\kappa^w$ | Wage flexibility | 0.03 |
| $\mu^p$ | SS price markup | 1 | $\phi_\pi$ | Taylor rule coefficient | 1.5 |
| $\mu^w$ | SS wage markup | 1 | $\rho_G$ | Persistence of gov. spending | 0.76 |
| $\delta$ | Depreciation rate | 0.08 | $\rho_B$ | Persistence of debt | 0.93 |

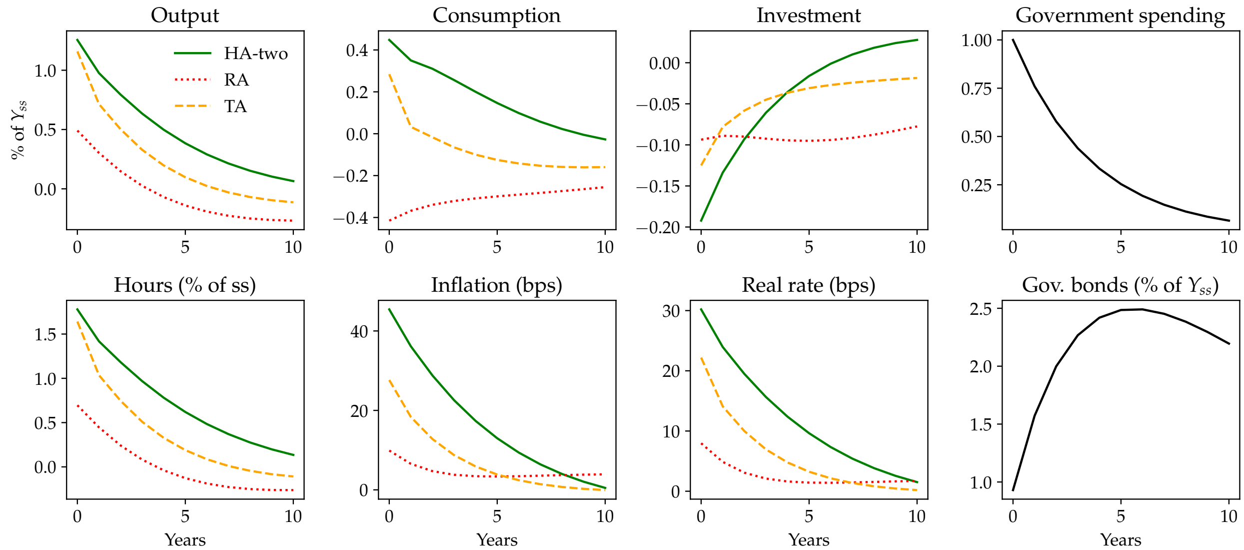
where $\zeta_t \equiv \int \left( (p_t + d_t)\, \varpi^{liq}_{it-1} + (1 + r_{t-1}) \left(1 - \varpi^{liq}_{it-1}\right)\right) a^{liq}_{it-1} di$; the bond market $\int (1 - \varpi_{i,t})\, a_{i,t} di = B_t$, and the equity market $\int \varpi_{i,t} a_{i,t} di = p_t$. (See appendix G.2 for a list of all the equilibrium equations.)

**Calibration.** Since the iMPC evidence that constitutes our primary calibration target is annual, we calibrate the entire model to an annual frequency. Our parameters and calibration targets are shown in table 3, with details in appendix G.3. We assume that the real interest rate is $r = 0.05$ and that the Frisch elasticity of labor supply is $\phi = 1$. For fiscal policy, we assume conventional targets for spending-to-GDP of $\frac{G}{Y} = 0.2$ and debt-to-GDP of $\frac{B}{Y} = 0.7$. For all our household models, we follow Kaplan and Violante (2022) and choose a ratio of aggregate assets to labor income of $\frac{A}{wN} = 4.2$. Assuming a labor share of income of $\frac{1-\alpha}{\mu^p} = 0.706$, this implies that aggregate assets to GDP are $\frac{A}{Y} = 4.2 \cdot 0.706 = 2.96$. Given that assets-to-GDP are given by $\frac{A}{Y} = \frac{B}{Y} + \frac{K}{Y} + \frac{1}{r}\left(1 - \frac{1}{\mu^p}\right)$, we have to distribute 2.26 GDPs between capital and price markups. We assume that $\mu^p \to 1$, so that all assets are capital. This implies a Cobb-Douglas coefficient on capital of $\alpha = 1 - \mu^p \cdot 0.706 = 0.294$, and a depreciation rate of $\delta = 0.08$, so that $K/Y = \alpha/(r + \delta) = 2.26$.

Next, we note that the ratio of post-tax to pre-tax labor income is $\frac{Z}{wN} = 1 - \frac{T}{wN}$, where the steady-state government budget constraint imposes $\frac{T}{wN} = \frac{\mu^p}{1-\alpha}\left(\frac{G}{Y} + r\frac{B}{Y}\right) = 0.33$. Hence, aggregate assets to post-tax income are $A/Z = 4.2/0.67 = 6.29$. For the six consumption-savings models that can match this target, we pick the exact same steady-state calibration as in section 4, as summarized in table 2. For wage markups, we make a parallel assumption to prices and set $\mu^w \to 1$; finally, we set the scale parameter in the disutility of labor supply, $\gamma$, to normalize output $Y$ to 1.

We set the parameters relevant to the dynamics as follows. We follow Auclert and Rognlie (2018) and assume an elasticity of investment to $Q$ of 4. We calibrate the slope of the Phillips curves using the standard formula implied by the Calvo model, $\kappa = \frac{1}{1+\Gamma}\left(1 - \frac{1}{1+r}(1 - \text{freq})\right) \text{freq}/(1 - \text{freq})$, where freq is the frequency of price change and $\Gamma$ is a real rigidity parameter. For prices, we convert the frequency of price change from Nakamura and Steinsson (2008) to annual, giving us freq $= 0.67$. For wages, we take Grigsby, Hurst and Yildirmaz (2021)'s freq $= 0.33$. We then apply to both a real rigidity coefficient of $\Gamma = 5$, delivering $\kappa^p = 0.23$ and $\kappa^w = 0.03$. This is

Figure 8: Government spending shock in quantitative HA-two model vs. RA and TA




*Note:* The HA-two model is a two-account heterogeneous-agent model which is calibrated to match evidence on intertemporal MPCs. The government spending shock declines exponentially at rate $\rho_G = 0.76$ and public debt follows $dB_t = \rho_B (dB_{t-1} + dG_t)$ for $\rho_B = 0.93$.

consistent with the conventional view in the literature that there is more rigidity in wages than prices. As our baseline, we follow Nakamura and Steinsson (2014)'s estimate of the persistence of military spending, which gives $\rho_G = 0.76$ at an annual level. For the persistence of debt, we use Auclert and Rognlie (2018)'s estimate of $\rho_B = 0.93$. While these numbers are subject to debate, we show that our results are robust to plausible changes all of these parameters. We solve the model numerically using the sequence-space Jacobian method of Auclert et al. (2021a).

### 7.2 Quantitative effect of deficit financed policy

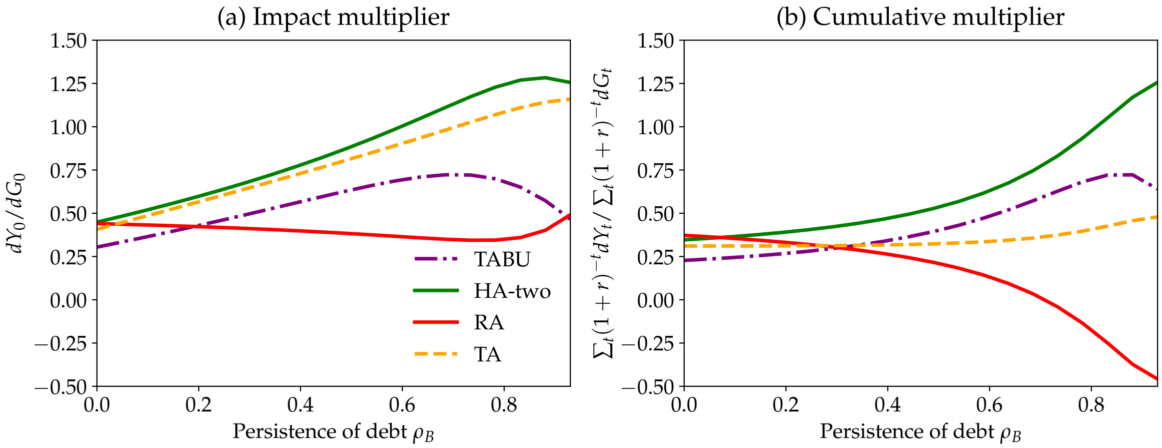
What are the effects of fiscal policy with capital, a realistic monetary policy rule, and realistic iMPCs? As we show next, the interaction between high intertemporal MPCs and primary deficits, which we emphasized in proposition 4, remains a crucial determinant of fiscal multipliers.

Figure 8 assumes a given degree of deficit financing ($\rho_B = 0.93$) and shows how iMPCs matter by comparing the responses of the RA, TA, and HA-two economies to the government spending shock. In all three models, the Taylor rule lifts real interest rates in response to the shock, crowding out investment. In the RA economy, consumption is crowded out as well, severely limiting the expansion of output. For TA, we find a stronger output response on impact, but one that fades quickly, driven by a fast reversal in consumption that mirrors primary deficits (cf proposition 6). HA-two stands out in that, despite the active Taylor rule and rising real rates, consumption contributes positively to output for about two years, offsetting investment crowd-out.

Figure 9 varies instead the degree of deficit financing $\rho_B$ across models.[48] Echoing our find-

[48]See figure G.1 in appendix G.4 for the full set of impulse responses.

Figure 9: Multipliers in the quantitative models




ings in section 5.4, HA-two and TABU predict impact and cumulative multipliers that increase in the degree of deficit financing. This is especially surprising as the RA model predicts cumulative multipliers that sharply *decline* with debt persistence: since inflation is forward-looking, pushing distortionary taxes to the future creates a larger inflation effect and therefore—given the active Taylor rule—more tightening by the central bank. The intertemporal Keynesian cross forces overcome this neoclassical effect. As in section 5.4, the TA model has impact multipliers that depend on deficit financing, but essentially flat cumulative multipliers.[49]

As expected given the active Taylor rule, the overall level of multipliers in figure 9 is significantly lower than in figure 5. Consistent with the results of section 5.4, however, the models share similar balanced-budget and cumulative multipliers, between 0.2 and 0.5. In the HA-two model, both impact and cumulative deficit-financed multipliers are around 1.3: this is the only model that generates deficit-financed multipliers above 1. TABU, on the other hand, has significantly lower multipliers.[50]

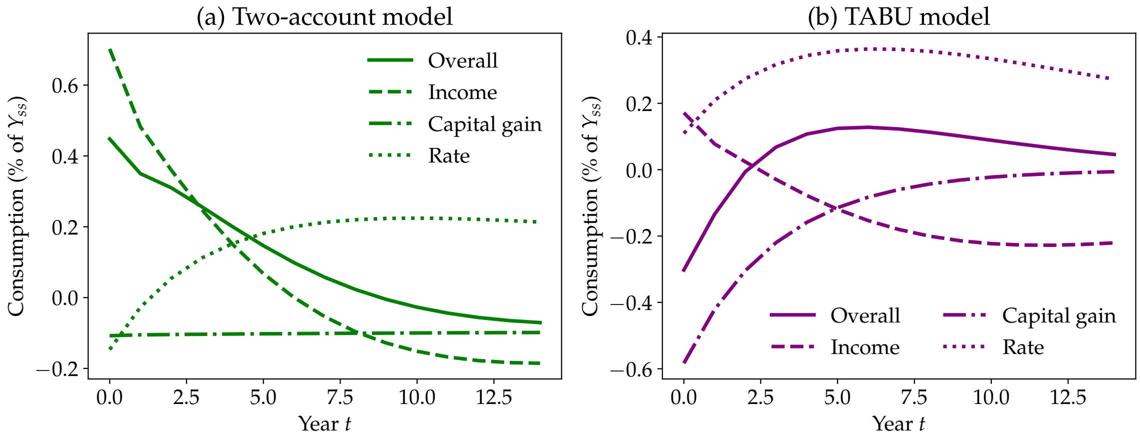
Recall from figure 5 that with the intertemporal Keynesian cross, TABU had higher multipliers than HA-two across the board. Why are these magnitudes flipped once real interest rates rise and investment falls? We can investigate this question by using the sequence-space decomposition in (34). This equation shows that the consumption response is the sum of responses to income; to an initial capital gain; and to a change in interest rates.

Figure 10 shows this decomposition for a deficit financed shock with $\rho_B = 0.93$ for HA-two and TABU. As figure 10(a) shows, the rise in consumption in HA-two is more than entirely accounted by the rise in labor income, consistent with the IKC, overcoming the crowding-out effect from rising real rates. Since investment is crowded out and interest rates rise, the stock market falls.

[49] In the TA model, inflation—and therefore the monetary response—is increased by future distortionary taxation as in the RA model, but is reduced by the negative demand effect of future tax increases, and these forces roughly balance out for the cumulative multiplier.

[50] Angeletos et al. (2023) further explore the effects of deficit-financed fiscal policy in a quantitative TABU model, finding that, under a fiscal rule that holds the tax rate fixed, "self-financing" can occur.

Figure 10: Decomposing the consumption responses in the two-account and TABU model




However, this capital loss is of limited consequence for consumption because, consistent with the data, the MPC out of capital gains is very low.

For the TABU model in figure 10(b), by contrast, this capital loss manifests itself in a strongly negative consumption response due to high MPCs out of capital gains $\mathbf{m}^{cap}$ (cf figure 7). This effect dampens aggregate demand, and hence the output response. With weaker output, the income component in figure 10(b) is also lower than for HA-two. Interestingly, the interest rate component actually contributes positively to consumption in TABU, even though real interest rates rise. This is a consequence of equation (35): with high $\mathbf{m}^{cap}$, $\mathbf{M}^{r}$ is high as well, and the income effect of interest rates starts to dominate.[51] This result suggests caution when using tractable analytical models for quantitative purposes: these models should be consistent not only with $\mathbf{M}$, but also $\mathbf{m}^{cap}$ from the data. To our knowledge, existing tractable models do not meet this bar.

To conclude, HA-two is unique among our models in generating positive consumption multipliers and an output multiplier above 1. As we demonstrate in appendix G.4, this result is robust to other parameter values, unless the Taylor rule coefficient is very high ($\phi_\pi \simeq 2$), the sensitivity of investment to $Q$ is very high ($\varepsilon_I \simeq 20$)—amplifying crowd-out—or wages and prices are very flexible, making neoclassical forces dominate the response to fiscal policy.

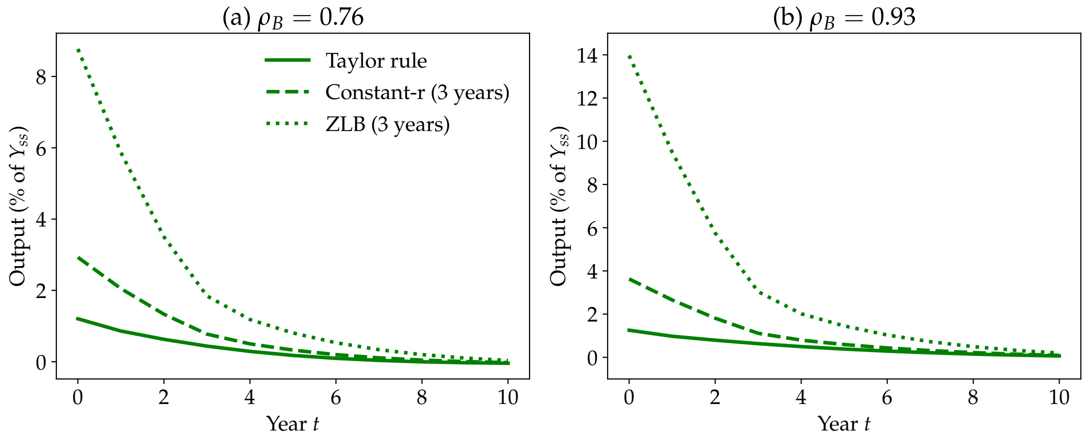
### 7.3 Role of monetary policy

So far we have parameterized the aggressiveness of monetary response with the conventional Taylor rule coefficient $\phi_\pi$, with higher $\phi_\pi$ amplifying the crowd-out effect. It is conventional in the literature to also study fiscal policy when nominal rates are immobilized by a zero lower bound (e.g. Christiano et al. 2011). We implement such a scenario by assuming a constant nominal interest rate for $T^{zlb} = 3$ years, with the economy reverting to the Taylor rule thereafter.

Figure 11 contrasts the impulse responses of output under the Taylor rule (solid) vs. the ZLB (dotted), at two levels of deficit financing. Irrespective of the degree of deficit financing, the ZLB

[51] See appendix G.4 for details on the quantitative TABU model and appendix F.4 for the extension of proposition 8.

Figure 11: Role of monetary policy: contrasting active Taylor rule, ZLB, and constant real rate




dramatically increases the output response, echoing the well-known result in the literature that the fiscal multiplier can be very large at the ZLB: positive inflation with unresponsive nominal interest rates means real interest rates fall on impact, crowding in both consumption and investment, generating further inflation, and so on.

The dashed line shows the effect of replacing our ZLB experiment with an experiment of maintaining the real interest rate constant for the same 3-year duration. Since now the nominal interest rate does increase in response to the inflationary effect of government spending, albeit less aggressively than with a Taylor rule, the outcome is somewhere in between the ZLB and the Taylor rule. This substantiates our claim in section 2 that a constant-$r$ monetary policy is an intermediate policy, standing between easy monetary policy, such as at the ZLB, and tight monetary policy, such as an active Taylor rule.

### 7.4 Taking stock

Table 4 revisits table 1 from the introduction and summarizes our main results for impact and cumulative multipliers under balanced budget fiscal policy $\rho_B = 0$ vs deficit financed policy $\rho_B = 0.93$. The two tables emphasize the *complementarity* between iMPCs and deficit financing: the combination of realistic iMPCs and deficit-financed fiscal policy predicts sizable multipliers above 1, both on impact and cumulatively. This is true with the IKC environment for TABU, HA-one and HA-two, and with our quantitative environment only for HA-two. HA-one cannot be calibrated to have reasonable aggregate assets, and TABU's iMPCs out of capital gains are far higher than is realistic (see our discussion in sections 6.3 and 7.2).

Multipliers for the quantitative HA-two model in table 4 lie between 0.3 and 1.3 depending on the horizon. The survey by Ramey (2019) concludes that the multiplier for temporary deficit-financed spending is "probably between 0.8 and 1.5", although reasonable people could argue that the data do not reject 0.5 or 2. There are two caveats, however, which complicate the comparison of our model-based conclusions with the data. First, in line with the theoretical literature, our

Table 4: Complementarity between iMPCs and deficit financing: multipliers across models

**IKC environment**

| Fiscal rule | Multiplier | RA | TA | TABU | HA-one | HA-two |
|---|---|---|---|---|---|---|
| **balanced budget** | impact | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| | cumulative | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| **deficit financing** | impact | 1.0 | 1.9 | 5.6 | 6.9 | 3.6 |
| | cumulative | 1.0 | 1.0 | 15.5 | 16.6 | 2.7 |

**Quantitative environment**

| Fiscal rule | Multiplier | RA | TA | TABU | HA-one | HA-two |
|---|---|---|---|---|---|---|
| **balanced budget** | impact | 0.4 | 0.4 | 0.3 | — | 0.4 |
| | cumulative | 0.4 | 0.3 | 0.2 | — | 0.3 |
| **deficit financing** | impact | 0.5 | 1.2 | 0.5 | — | 1.3 |
| | cumulative | -0.5 | 0.5 | 0.6 | — | 1.3 |

*Note.* Simulated multipliers. Columns present five models of household behavior, whose calibrations are presented in table 2. Panels describe two different general equilibrium environments: the "IKC environment" described in sections 2–5 with only government spending and bonds; and the "quantitative environment" described in section 7, which includes capital and a Taylor rule, The government spending shock is an AR(1) with (annual) persistence $\rho_G = 0.76$. "Balanced budget" corresponds to contemporaneous taxation, and "deficit financing" assumes a persistence parameter of public debt of $\rho_B = 0.93$. HA-one cannot match the level of aggregate assets of our quantitative environment.

economy was assumed to be entirely closed; openness should reduce multipliers somewhat as it dampens the feedback between consumption and income (see Aggarwal et al. 2023). Second, the empirical literature typically characterizes a single type of "multiplier"; according to our model, however, the degree of deficit financing matters greatly for multipliers. This heavy dependence of cumulative multipliers on deficit financing constitutes a new testable prediction for the empirical literature. Third, we have mostly restricted our attention to fiscal policies which adjust income taxes to raise tax revenues, without altering tax progressivity. However, as we discuss in section 5.1, the precise tax instruments used can be crucial, with less progressive taxation reducing multipliers.

## 8 Conclusion

In this paper, we derive an intertemporal version of the static Keynesian cross and use it to analyze the general equilibrium effects of fiscal policy. In the intertemporal Keynesian cross (IKC), the interaction between deficit financing and intertemporal marginal propensities to consume (iMPCs) determines fiscal multipliers. This holds exactly in the environment of section 2, and numerically in a more quantitative environment, provided that we also match the response of consumption to capital gains in the data. We provide empirical estimates of intertemporal MPCs and find that, within a set of commonly used models, only a heterogeneous-agent model with two accounts can match these estimates. Quantitatively, this model suggests that fiscal multipliers with deficit financing are strictly above 1.

Our paper provides a new approach to studying models with heterogeneity through the use of sequence-space Jacobians. Moving beyond the literature on sufficient statistics in partial equilibrium, we reduce the complexity of *general* equilibrium to a matrix of sufficient statistics, intertemporal MPCs, that can be disciplined empirically. This approach might be fruitfully extended to many other areas in macroeconomics, since the key insight—that agents interact in general equilibrium through a limited set of aggregates—applies to a wide variety of models.

### Data availability

Data and code replicating all the tables, figures, and other results in this paper can be found in Auclert, Rognlie and Straub (2024a), available in the Harvard Dataverse at `https://doi.org/10.7910/DVN/AKICUR`.

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# The Intertemporal Keynesian Cross
# Online Appendix

Adrien Auclert, Matthew Rognlie and Ludwig Straub

## A Appendix to section 2

### A.1 Primitives of the consumption function

In this section, we formally describe the class of models of consumption and saving behavior considered in section 2.2. Household have a potentially multidimensional exogenous discrete state $\mathbf{s}$, which follows a Markov process $\Pi(\mathbf{s})$, and whose initial distribution $\pi(\mathbf{s})$ across households is a stationary distribution of $\Pi$. Households' units of effective labor $e(\mathbf{s})$ depend on $\mathbf{s}$, so the Markov process can represent both permanent differences in income as well as income shocks. The utility function can also depend on $\mathbf{s}$, representing preference shocks. Constraints on asset trade can also depend on $\mathbf{s}$.

At date $t$, households $i$ choose their continuous state $\boldsymbol{\omega}_{it} = (c_{it}, a_{it}^1, \ldots a_{it}^L, \omega_{it}^{L+1}, \ldots, \omega_{it}^K)$, a $K$-dimensional vector that includes consumption today, $c_{it}$, asset holdings $a_{it}^1, \ldots, a_{it}^L$ between $t$ and $t+1$ in $L \geq 1$ different accounts, and potentially other continuous states $\omega_{it}^{L+1}, \ldots, \omega_{it}^K$, with $K \geq L+1$. They do so to maximize utility $u(\mathbf{s}_{it}, \boldsymbol{\omega}_{it-1}, \boldsymbol{\omega}_{it}, \mathbf{X}_t)$, subject to a budget constraint across all assets and an arbitrary additional constraint $\boldsymbol{\omega}_{it} \in \Gamma(\mathbf{s}_{it}, \boldsymbol{\omega}_{it-1}, \mathbf{X}_t)$. Utility $u$ and the constraint $\mathcal{G}$ also take in the current exogenous state $\mathbf{s}_{it}$, the lagged endogenous state $\boldsymbol{\omega}_{it-1}$, and a vector $\mathbf{X}_t$ of relevant aggregate variables, which includes $(Y_t, T_t, r_{t-1})$ and possibly other aggregates.

The resulting Bellman equation at $t$ is therefore:

$$\begin{aligned} V_t(\mathbf{s}_{it}, \boldsymbol{\omega}_{it-1}) &= \max_{\boldsymbol{\omega}_{it}} u(\mathbf{s}_{it}, \boldsymbol{\omega}_{it-1}, \boldsymbol{\omega}_{it}, \mathbf{X}_t) + \beta \mathbb{E}\left[V_{t+1}(\mathbf{s}_{it+1}, \boldsymbol{\omega}_{it}) | \mathbf{s}_{it}\right] \\ &\text{s.t.} \quad c_{it} + \sum_{l=1}^{L} a_{it}^l = (1 + r(\mathbf{X}_t)) \sum_{l=1}^{L} a_{it-1}^l + z_{it}(\mathbf{s}_{it}, \mathbf{X}_t) \\ &\quad\quad \boldsymbol{\omega}_{it} \in \Gamma(\mathbf{s}_{it}, \boldsymbol{\omega}_{it-1}, \mathbf{X}_t) \end{aligned} \tag{A.1}$$

where $z_{it}(\mathbf{s}_t, \mathbf{X}_t)$ is after-tax income for the individual in state $\mathbf{s}_{it}$ at time $t$.

Note that the budget constraint across all assets in (A.1) can be rewritten as (9) from the main text,

$$c_{it} + a_{it} = (1 + r(\mathbf{X}_t)) a_{it-1} + z_{it}(\mathbf{s}_t, \mathbf{X}_t)$$

given the substitution $a_{it} \equiv \sum_{l=1}^{L} a_{it}^l$.

The solution to (A.1) involves a time-dependent policy function $\boldsymbol{\omega}_t(\mathbf{s}, \boldsymbol{\omega}_-)$, and associated consumption policy $c_t(\mathbf{s}, \boldsymbol{\omega}_-)$. Denote by $\mu_t(\mathbf{s}, \boldsymbol{\omega}_-)$ the distribution of agents across states $(\mathbf{s}, \boldsymbol{\omega}_-)$ at the beginning of period $t$. The law of motion of the distribution is given by

$$\mu_{t+1}\left(\mathbf{s}',\Omega\right)=\sum_{\mathbf{s}}\Pi\left(\mathbf{s},\mathbf{s}'\right)\int\mathbf{1}_{\{\omega_t(\mathbf{s},\omega_-)\in\Omega\}}d\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right) \tag{A.2}$$

for all $\mathbf{s}'$ and all sets $\Omega$ to which $\boldsymbol{\omega}$ can belong. We denote by $a_{it}\equiv\sum_{l=1}^{L}a_{it}^{l}$ the aggregate asset position of agents. Then, the budget constraint in (A.1) is just (9), which is the key restriction across the models in this section.

We are interested in the time path of $J\geq 2$ aggregate outcomes $\{\mathbf{Y}_t\}\equiv\{C_t,A_t,Y_{3t}\ldots,Y_{Jt}\}$. For each $j=1\ldots J$, we define the individual outcome $y_j\left(\mathbf{s},\boldsymbol{\omega}_-,\boldsymbol{\omega},\mathbf{X}\right)$ as some function of individual states, policies, and inputs, that has a bounded derivative. The aggregate outcome $Y_{jt}$ is then defined as the aggregated individual outcome,

$$Y_{jt}\equiv\int y_j\left(\mathbf{s},\boldsymbol{\omega}_-,\boldsymbol{\omega}_t\left(\mathbf{s},\boldsymbol{\omega}_-\right),\mathbf{X}_t\right)d\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right) \tag{A.3}$$

The two outcomes that we always keep track of are individual consumption $c_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ and individual assets $a_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)=\sum_{l=1}^{L}a_t^l\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ (recall that $c_t$ and $a_t^l$ are part of $\boldsymbol{\omega}_t$). Aggregate consumption and aggregate assets defined as:

$$\begin{aligned}C_t &\equiv \int c_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)d\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)\\ A_t &\equiv \int a_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)d\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)\end{aligned}$$

**Steady-state and transition given aggregate inputs $\{\mathbf{X}_t\}$ and outcomes $\{\mathbf{Y}_t\}$.**

In a *steady state*, $\mathbf{X}_t$ is a constant $\mathbf{X}$, the policy is a constant $\boldsymbol{\omega}\left(\mathbf{s},\boldsymbol{\omega}_-\right)$, and $\mathbf{Y}_t$ is also a constant $\mathbf{Y}$. We assume a unique stationary measure $\mu\left(\mathbf{s},\boldsymbol{\omega}_-\right)$, solving (A.2) given the steady state $\boldsymbol{\omega}\left(\mathbf{s},\boldsymbol{\omega}_-\right)$.

Given a perfect-foresight sequence $\{\mathbf{X}_t\}$, we define a *transition* as:

a) the time-dependent policies $\boldsymbol{\omega}_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ and value function $V_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ that solve the Bellman equation (A.1)

b) the time-varying measure $\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ that solves (A.2), starting from the initial measure $\mu_0\left(\mathbf{s},\boldsymbol{\omega}_-\right)=\mu\left(\mathbf{s},\boldsymbol{\omega}_-\right)$

Finally, we define the *aggregate outcome functions* $\mathcal{Y}_{jt}\left(\{\mathbf{X}_t\}\right)$ as

$$\mathcal{Y}_{jt}\left(\{\mathbf{X}_t\}\right)\equiv\int y_j\left(\mathbf{s},\boldsymbol{\omega}_-,\boldsymbol{\omega}_t\left(\mathbf{s},\boldsymbol{\omega}_-\right),\mathbf{X}_t\right)d\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right) \tag{A.4}$$

through the dependence of $\boldsymbol{\omega}_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$, $\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ and $\mathbf{X}_t$ on $\{\mathbf{X}_t\}$ in a transition.

In particular, we define the consumption function $\mathcal{C}_t\left(\{\mathbf{X}_t\}\right)$ and the aggregate asset function $\mathcal{A}_t\left(\{\mathbf{X}_t\}\right)$ as in (A.4), through the dependence of $c_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$, $a_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ and $\mu_t\left(\mathbf{s},\boldsymbol{\omega}_-\right)$ on $\{\mathbf{X}_t\}$. Note that the budget constraint (9) implies that we must have for all $t$:

$$\mathcal{C}_t\left(\{\mathbf{X}_t\}\right)+\mathcal{A}_t\left(\{\mathbf{X}_t\}\right)=\left(1+r\left(\mathbf{X}_t\right)\right)\mathcal{A}_{t-1}\left(\{\mathbf{X}_t\}\right)+Y_t-T_t \tag{A.5}$$

since aggregate taxes are defined as $\int z_{it}(\mathbf{s}_t, \mathbf{X}_t)\, di = Y_t - T_t$ by the definition of taxes.

**Consumption function for the special case of section 2.2.** In the environment described in section 2.2, $\mathbf{X}_t = Z_t$ is single dimensional, and only enters the Bellman (A.1) through $z_{it}(\mathbf{s}_t, Z_t) = e(\mathbf{s}_{it})^{1-\theta} / \sum_{\tilde{\mathbf{s}}} \pi(\tilde{\mathbf{s}})\, e(\tilde{\mathbf{s}})^{1-\theta} Z_t$. This delivers $\mathcal{C}_t(\{Z_s\})$, ie equation (10). In this environment, (A.5) reads simply:

$$\mathcal{C}_t(\{Z_s\}) + \mathcal{A}_t(\{Z_s\}) = (1+r)\,\mathcal{A}_{t-1}(\{Z_s\}) + Z_t \tag{A.6}$$

## A.2 Implementing a constant-real interest rate rule

This section describes in more detail how monetary policy operates in our model.

Nominal one-period bonds in zero net supply (or "reserves"), with nominal return $i_t$, can be traded by the central bank and financial market arbitrageurs, but they cannot be held by agents directly. Financial market arbitrageurs can freely invest in nominal bonds $B_t$ and real bonds $b_t$. Their nominal profits in period $t$ are given by

$$\Pi_t = (1 + i_{t-1})\, B_{t-1} + (1 + r_{t-1})\, P_t b_{t-1} - B_t - P_t b_t$$

Arbitrageurs maximize the present discounted value of $\Pi_t$ at the sequence of nominal interest rates $i_t$. Optimization immediately implies the Fisher equation:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{A.7}$$

for all $t \geq 0$. Since nominal bonds are in zero supply, market clearing imposes $B_t = 0$ for all arbitrageurs. We also assume that they start with zero assets, so they have $b_t = 0$ at all times. Hence, arbitrageur profits are zero at all times and we can omit them from the model.

The central bank directly sets the interest rate $i_t$ on reserves. Given a sequence $r_t^{exo}$, it does so according to the rule

$$1 + i_t = (1 + r_t^{exo})\,(1 + \pi_{t+1}) \tag{A.8}$$

Combining (A.7) and (A.8) immediately implies $r_t = r_t^{exo}$ for all $t \geq 0$. In section 2.2, we further assume that $r_t^{exo} = r$, the steady state real interest rate; and then consider an arbitrary exogenous path for $r_t^{exo}$ in section 2.5.

An alternative to this setup with arbitrageurs would be to let agents trade both nominal and real bonds, and assume that initial portfolios are such all agents only hold real bonds. This route implies a more complex formulation of the household problem, so we do not pursue it here. Appendix B.2 considers a modification of the model where all agents hold only nominal bonds instead, as in Auclert (2019), Angeletos et al. (2023) and Kaplan, Nikolakoudis and Violante (2023).

## A.3 Wage Phillips curve

In this section, we derive a wage Phillips curve for our environment by adapting the standard microfoundation in the New Keynesian sticky-wage literature (Erceg et al. 2000, Schmitt-Grohé and Uribe 2005) to a heterogeneous-agent environment.

We assume that labor hours $n_{it}$ are determined by union labor demand. Every worker $i$ belongs to a union $k$. There are a continuum of such unions, and they each hire a fully representative sample of the population. Each union $k$ aggregates the efficient units of work of its members into a union-specific task $N_{kt} = \int e_{it} n_{ikt} di$. A competitive labor packer then packages these tasks into aggregate employment services using the constant-elasticity-of-substitution technology

$$N_t = \left( \int_k N_{kt}^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

and sells these services to final goods firms at price $W_t$.

We assume that there are quadratic utility costs of adjusting the nominal wage $W_{kt}$ set by union $k$, by allowing for an extra additive disutility term $\frac{\psi}{2} \int_k \left( \frac{W_{kt}}{W_{kt-1}} - 1 \right)^2 dk$ in the household flow utility term of the Bellman equation (A.1). In every period $t$, we restrict each union $k$ to set a common wage $W_{kt}$ per efficient unit for each of its members, and to call upon its members to supply hours according to a uniform rule, so that $n_{ikt} = N_{kt}$. Given these two restrictions, the union sets $W_{kt}$ to maximize the average utility of its members.

In this setup, all unions choose to set the same wage $W_{kt} = W_t$ at time $t$ and all households work the same number of hours, equal to $n_{it} = N_t$, so efficiency-weighted hours worked $\int e_{it} n_{it} di$ are also equal to aggregate labor demand $N_t$. Hence, this delivers the labor market setup of section 2.2.

At any time $t$, union $k$ sets its wage $W_{kt}$ to maximize, on behalf of all the workers it employs,

$$\sum_{\tau \geq 0} \beta^{t+\tau} \left( \int \left\{ u\left(c_{it+\tau}\right) - v\left(n_{it+\tau}\right) \right\} d\Psi_{it+\tau} - \frac{\psi}{2} \left( \frac{W_{k,t+\tau}}{W_{k,t+\tau-1}} - 1 \right)^2 \right)$$

taking as given the initial distribution of households over idiosyncratic states $\Psi_{it}$ as well as the demand curve for tasks emanating from the labor packers, which is

$$N_{kt} = \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon} N_t \tag{A.9}$$

where $W_t = \left( \int W_{kt}^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}}$ is the price index for aggregate employment services.

Consider household $i$, working for union $k$. By (8), their total real earnings are

$$\begin{aligned} z_{it} &= \tau_t \left( \frac{W_{kt}}{P_t} e_{it} N_{kt} \right)^{1-\theta} \\ &= \tau_t \left( \frac{e_{it}}{P_t} W_{kt} \left( \frac{W_{kt}}{W_t} \right)^{-\varepsilon} N_t \right)^{1-\theta} \end{aligned}$$

The envelope theorem applied to the problem in (A.1) implies that we can evaluate indirect utility as if all income from the union wage change is consumed. In that case, $\frac{\partial c_{it}}{\partial W_{kt}} = \frac{\partial z_{it}}{\partial W_{kt}}$, where, exploiting the fact that in equilibrium $W_{kt} = W_t$, we have:

$$\begin{aligned} \frac{\partial z_{it}}{\partial W_{kt}} &= (1-\theta)\,\tau_t \left( \frac{W_{kt}}{P_t} e_{it} N_{kt} \right)^{-\theta} \frac{e_{it}}{P_t} \left\{ N_{kt} - W_{kt}\varepsilon \left( \frac{1}{W_t} \right)^{-\varepsilon} N_t W_{kt}^{-\varepsilon-1} \right\} \\ &= (1 - MTR_{it}) \frac{e_{it}}{P_t} N_{kt} (1-\varepsilon) \end{aligned}$$

where $MTR_{it} \equiv 1 - (1-\theta)\,\tau_t \left( \frac{W_{kt}}{P_t} e_{it} N_{kt} \right)^{-\theta}$ is household $i$'s marginal tax rate at time $t$. On the other hand, household $i$'s total hours worked $n_{it}$ are given by (A.9), so they satisfy

$$\frac{\partial n_{it}}{\partial W_{kt}} = -\varepsilon \frac{N_{kt}}{W_{kt}}$$

The first-order condition of the union with respect to $W_{kt}$ is therefore

$$\begin{aligned} &\int N_{kt} \left\{ (1-\varepsilon) \frac{e_{it}}{P_t} u'(c_{it}) (1 - MTR_{it}) + \frac{\varepsilon}{W_{kt}} v'(n_{it}) \right\} d\Psi_{it} \\ &-\psi \left( \frac{W_{k,t}}{W_{k,t-1}} - 1 \right) \frac{1}{W_{k,t-1}} + \beta\psi \left( \frac{W_{k,t+1}}{W_{k,t}} - 1 \right) \left( \frac{W_{k,t+1}}{W_{k,t}} \right) \frac{1}{W_{k,t}} = 0 \end{aligned} \tag{A.10}$$

In equilibrium all unions set the same wage, so $W_{kt} = W_t$ and $N_{kt} = N_t$. Define wage inflation $\pi^w \equiv \frac{W_t}{W_{t-1}} - 1$. After multiplying (A.10) by $W_t$, and noting that

$$\frac{\partial z_{it}}{\partial n_{it}} = (1 - MTR_{it})\, e_{it} \frac{W_t}{P_t}$$

we find that aggregate nominal wage inflation $1 + \pi_t^w \equiv \frac{W_t}{W_{t-1}}$ is described by the following non-linear New Keynesian Phillips Curve:

$$\pi_t^w (1 + \pi_t^w) = \frac{\varepsilon}{\psi} \int N_t \left( v'(n_{it}) - \frac{\varepsilon - 1}{\varepsilon} \frac{\partial z_{it}}{\partial n_{it}} u'(c_{it}) \right) di + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w) \tag{A.11}$$

According to (A.11), conditional on future wage inflation, unions set higher nominal wages when an average of marginal rates of substitution between hours and consumption for households $v'(n_{it}) / u'(c_{it})$ exceeds a marked-down average of marginal after-tax income from extra hours

$\frac{\partial z_{it}}{\partial n_{it}}$.[A-1]

Note that in equilibrium, enforcing $n_{it} = N_{kt} = N_t$, we further have that

$$\frac{\partial z_{it}}{\partial n_{it}} = (1-\theta)\,\tau_t e_{it}^{1-\theta}\left(\frac{W_t}{P_t}\right)^{1-\theta} N_t^{-\theta} = (1-\theta)\,\frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta}di}\frac{Z_t}{N_t}$$

where $Z_t$ is average after-tax income. Hence equation (A.11) can be written in terms of aggregates $\pi_t^w$, $Z_t$, $N_t$ together with a "virtual aggregate consumption" term $C_t^*$

$$\pi_t^w(1+\pi_t^w) = \frac{\varepsilon}{\psi}\left\{N_t v'(N_t) - \frac{\varepsilon-1}{\varepsilon}(1-\theta)\,Z_t u'(C_t^*)\right\} + \beta\pi_{t+1}^w(1+\pi_{t+1}^w)$$

where we have defined $C_t^*$ such

$$u'(C_t^*) = \int_i \frac{e_{it}^{1-\theta}u'(c_{it})}{\int e_{it}^{1-\theta}di}di$$

Hence, in this setting, the distribution matters for inflation dynamics only through its effects on the dynamics of $C_t^*$. Linearizing this expression around the zero inflation steady state yields a standard wage Phillips Curve

$$\pi_t^w = \kappa^w\left\{\frac{1}{\phi}\frac{dN_t}{N} + \frac{1}{\sigma}\frac{dC_t^*}{C^*} - \left(\frac{dZ_t}{Z} - \frac{dN_t}{N}\right)\right\} + \beta\pi_{t+1}^w \tag{A.12}$$

where $\kappa^w \equiv \frac{\varepsilon}{\psi}Nv'(N)$, $\phi$ is the Frisch elasticity of labor supply, and $\sigma$ the elasticity of intertemporal substitution in consumption.[A-2] The term $\frac{dZ_t}{Z} - \frac{dN_t}{N}$ captures the distortionary effects of taxation.

## A.4 Infinite matrix representation of the derivative of the consumption function

We first define what it means for a linear operator on a sequence space $\ell^p$ to be represented by an infinite matrix.

**Definition 1** (Operator represented by matrix)**.** We say that a linear operator $\mathbf{M}$ on $\ell^p$, with $1 \leq p \leq \infty$, *can be represented by the infinite matrix* $[M_{ts}]_{t,s=0}^{\infty}$ if for all $\mathbf{x} = (x_0, x_1, \ldots) \in \ell^p$, defining

[A-1] This term includes the distortions from labor income taxes, which are important for fiscal multipliers (Uhlig 2010).

[A-2] To see where (A.12) comes from, assume for simplicity that $v$ and $u$ have constant elasticities, so that $v'(N) = N^{\frac{1}{\phi}}$ and $u'(C^*) = (C^*)^{-\frac{1}{\sigma}}$. Then, linearizing the term in brackets, we obtain

$$\begin{aligned}
&\left(1+\frac{1}{\phi}\right)N^{1+\frac{1}{\phi}}dN_t - \frac{\varepsilon-1}{\varepsilon}(1-\theta)\left(dZ_t\,(C^*)^{-\sigma} - \frac{1}{\sigma}Z\,(C^*)^{-\sigma-1}dC_t^*\right)\\
&= \left(1+\frac{1}{\phi}\right)N^{1+\frac{1}{\phi}}\frac{dN_t}{N} - \frac{\varepsilon-1}{\varepsilon}(1-\theta)\,Z\,(C^*)^{-\sigma}\left(\frac{dZ_t}{Z} - \frac{1}{\sigma}\frac{dC_t^*}{C^*}\right)\\
&= Nv'(N)\left\{\left(1+\frac{1}{\phi}\right)\frac{dN_t}{N} - \left(\frac{dZ_t}{Z} - \frac{1}{\sigma}\frac{dC_t^*}{C^*}\right)\right\}
\end{aligned}$$

where we have exploited the fact that, in the zero inflation steady state, $N^{1+\frac{1}{\phi}} = \frac{\varepsilon-1}{\varepsilon}(1-\theta)\,Z\,(C^*)^{-\sigma}$.

$\mathbf{y} = \mathbf{M}\mathbf{x} = (y_0, y_1, \ldots) \in \ell^p$, we have for all $t = 0 \ldots \infty$

$$y_t = \sum_{s=0}^{\infty} M_{ts} x_s \tag{A.13}$$

This definition extends the standard notion, in finite dimensions, of a matrix representing a linear transformation (see e.g. Horn and Johnson 2012), by taking an infinite rather than a finite sum. For general $\ell^p$, a discussion is available in Böttcher and Grudsky (2005). For $\ell^2$ specifically, and Hilbert spaces more generally, see e.g. Halmos (1982) and Conway (2007).[A-3]

It is simple to show that for any bounded linear $\mathbf{M} : \ell^p \to \ell^p$, as long as $1 \leq p < \infty$, there is a unique matrix representation.[A-4] For an arbitrary bounded linear $\mathbf{M} : \ell^\infty \to \ell^\infty$, however, existence of a matrix representation in the sense of definition 1 is not guaranteed. We discuss a non-economic counterexample at the end of this section.

Even so, we are able to verify that, for all the economic models introduced in section 4, $\mathbf{M}$ does have a matrix representation. For our analytical models, we do so by explicitly constructing the matrix. For our heterogenous-agent quantitative models, we numerically verify the sufficient condition described below, and also the stronger stationarity condition in Auclert et al. (2023b), which implies a matrix representation with a specific "quasi-Toeplitz" structure.

**Sufficient condition for infinite matrix representation.**

**Lemma 2.** *Consider a bounded linear operator $\mathbf{M} : \ell^\infty \to \ell^\infty$. Suppose that there are two constants $K > 0$ and $\gamma \in (0, 1)$, such that for any $\mathbf{x}^\tau \in \ell^\infty$ that has entries of 0 before the $\tau$th entry, we have*

$$|(\mathbf{M}\mathbf{x}^\tau)_t| < K\gamma^{\tau - t} \|\mathbf{x}^\tau\| \tag{A.14}$$

*Then the infinite matrix $[M_{ts}]_{t,s=0}^\infty$ defined by $M_{ts} \equiv (\mathbf{M}\mathbf{e}_s)_t$ represents $\mathbf{M}$.*

*Proof.* For arbitrary $\mathbf{x} = \{x_t\}_{t=0}^\infty \in \ell^\infty$ and any index $\tau$, we can write

$$\mathbf{x} = x_0\mathbf{e}_0 + x_1\mathbf{e}_1 + \ldots + x_{\tau-1}\mathbf{e}_{\tau-1} + \mathbf{x}^\tau$$

where $\mathbf{e}_t \in \ell^\infty$ is the sequence with 0s everywhere except a 1 at $t$, and $\mathbf{x}^\tau \in \ell^\infty$ is the sequence with 0s prior to $\tau$ and the same entries $x_\tau, x_{\tau+1}, \ldots$ as $\mathbf{x}$ from $\tau$ on. By linearity we can write

$$\mathbf{M}\mathbf{x} = \mathbf{M}(x_0\mathbf{e}_0) + \mathbf{M}(x_1\mathbf{e}_1) + \ldots + \mathbf{M}(x_{\tau-1}\mathbf{e}_{\tau-1}) + \mathbf{M}\mathbf{x}^\tau$$

[A-3] Depending on their point of view, authors in these references may talk about matrices "representing" operators, a "correspondence" between matrices and operators, or operators "induced by" matrices.

[A-4] The argument is as follows: for any $\mathbf{x} \in \ell^p$ with $1 \leq p < \infty$, we can write $\mathbf{x} = \sum_{s=0}^\infty x_s\mathbf{e}_s$, with convergence following from $\left\| x - \sum_{s=0}^T x_s\mathbf{e}_s \right\| = \left(\sum_{s=T+1}^\infty |x_s|^p\right)^{1/p} \to 0$ as $T \to \infty$. Defining $M_{ts} \equiv (\mathbf{M}\mathbf{e}_s)_t$, for any $\mathbf{x}$ we can write $(\mathbf{M}\mathbf{x})_t = (\mathbf{M}\sum_{s=0}^\infty x_s\mathbf{e}_s)_t = \sum_{s=0}^\infty x_s(\mathbf{M}\mathbf{e}_s)_t = \sum_{s=0}^\infty M_{ts}x_s$, verifying (A.13). Note that interchanging $\mathbf{M}$ and the infinite sum is possible because $\mathbf{M}$ is bounded and linear, and uniqueness follows from applying (A.13) to $\mathbf{x} = \mathbf{e}_s$.

Then, defining $M_{ts} \equiv (\mathbf{M}\mathbf{e}_s)_t$ and using the inequality (A.14), we can write

$$|(\mathbf{M}\mathbf{x})_t - (M_{t0}x_0 + \ldots + M_{t\tau-1}x_{\tau-1})| = |(\mathbf{M}\mathbf{x}^\tau)_t| < K\gamma^{\tau-t}\|\mathbf{x}^\tau\| \leq K\gamma^{\tau-t}\|\mathbf{x}\|$$

and taking the limit as $\tau \to \infty$, $\gamma^{\tau-t} \to 0$, implying that $\sum_{s=0}^{\infty} M_{ts}x_s$ converges to $y_t = (\mathbf{M}\mathbf{x})_t$, as required by definition 1. □

For our application, where $\mathbf{M}$ is the derivative of the consumption function, condition (A.14) is intuitive. It says that the effect of a bounded perturbation to aggregate after-tax income—a perturbation that begins at some future date $\tau$—on aggregate consumption at an earlier date $t$ must eventually go to zero as the horizon $\tau - t$ increases, at some rate $\gamma < 1$.[A-5]

One illustration of condition (A.14) is the following. Note that the present value of the absolute value of $\mathbf{x}^\tau$ at date $\tau$ is bounded by $\frac{1+r}{r}\|\mathbf{x}^\tau\|$ (given our assumption in this paper that $r > 0$). Suppose that the effect of any $\mathbf{x}^\tau$ on consumption at an earlier date $t$ is bounded by some arbitrarily large constant multiple $K_0$ of the present value in date-$t$ terms.[A-6] Then (A.14) holds with $K = K_0\frac{1+r}{r}$ and $\gamma = \frac{1}{1+r}$.

We can verify that this condition holds for all our analytical models directly: for instance, in the TA consumption function (A.61), the impact of a shock to future income on consumption today is exactly the present value of the shock times $(1-\mu)\frac{r}{1+r}$. In appendix D, we go further and derive the exact infinite matrix representations of all these analytical models.

**Sufficient condition for matrix representation in quantitative models.** On the computer, a discretized representation of the quantitative model will be used. Following Auclert et al. (2021a) and Auclert et al. (2023b), we use the following notation to represent the equations of the household problem described in section A.1:

$$\mathbf{v}_t = v(\mathbf{v}_{t+1}, Z_t) \tag{A.15}$$

$$\mathbf{D}_{t+1} = \Lambda(\mathbf{v}_{t+1}, Z_t)'\mathbf{D}_t \tag{A.16}$$

$$C_t = c(\mathbf{v}_{t+1}, Z_t)'\mathbf{D}_t \tag{A.17}$$

Equation (A.15) is a discretized representation of the value function, equation (A.16) a discretized representation of the law of motion of the distribution, and equation (A.17) a discretized representation of the aggregate consumption function defined in (A.4). We then have the following

[A-5] In fact, it is possible to generalize the lemma slightly, replacing $\gamma^{\tau-t}$ by an arbitrary sequence $k_{\tau-t} \to 0$ .

[A-6] This is a weak condition: it says that if aggregate income is perturbed in the future, that cannot cause consumption today at $t$ to increase by more than some multiple of the present value at $t$ of the absolute perturbation to income. Indeed, if consumption never responds negatively at any date to a positive income perturbation, the condition holds with $K = 1$: intertemporal budget balance is violated if consumption at $t$ increases by more than the date-$t$ present value of the income increase, unless there is some offsetting negative response to the income increase elsewhere. To allow for the (unusual) more general case where consumption can respond negatively to income increases, we relax to arbitrarily large $K > 1$.

representation result.[A-7]

**Proposition 9.** *Assume that $v$, $\Lambda$, and $c$ are continuously differentiable around the steady state and that*

*a) The derivative $\mathbf{v_v}$ of $v$ wrt its first argument around the steady state has spectral radius strictly less than 1, which we denote by $\beta$*

*b) The steady state transition matrix $\Lambda$ has spectral radius of at most 1*

*Then, the Frechet derivative of $C_t$ with respect to $Z_t$, $\mathbf{M}$, satisfies the condition of lemma 2. $\mathbf{M}$ can therefore be represented by the infinite matrix with elements $M_{ts} \equiv (\mathbf{M}\mathbf{e}_s)_t$.*

*Proof.* The proof has four main steps.

*Step 1.*

We first argue that (A.15) implies a Frechet differentiable mapping $\mathcal{V}$ from $(Z_0, Z_1, \ldots)$ to $(\mathbf{v}_0, \mathbf{v}_1, \ldots)$ defined on a neighborhood around the steady state. To do so, we use the implicit function theorem for Banach spaces (with the sup norm, as always in this paper) on the map $\bar{\mathcal{V}}$ from $(Z_0, Z_1, \ldots) \times (\mathbf{v}_0, \mathbf{v}_1, \ldots)$ to $(\mathbf{v}_0 - v(\mathbf{v}_1, Z_0), \mathbf{v}_1 - v(\mathbf{v}_2, Z_1), \ldots)$. This requires continuous differentiability of $\bar{\mathcal{V}}$, which follows from the continuous differentiability of $v$, and also that the derivative of $\bar{\mathcal{V}}$ with respect to $(\mathbf{v}_0, \mathbf{v}_1, \ldots)$, which is the linear map $(d\mathbf{v}_0, d\mathbf{v}_1, \ldots) \to (d\mathbf{v}_0 - \mathbf{v_v} d\mathbf{v}_1, d\mathbf{v}_1 - \mathbf{v_v} d\mathbf{v}_2, \ldots)$, is a Banach space isomorphism. Given the bounded inverse theorem, this follows from bijectivity of the linear map, which is straightforward to show.[A-8]

*Step 2.*

It follows from differentiability of $\mathcal{V}$ that if there is a perturbation $d\mathbf{Z}^\tau = \mathbf{x}^\tau$ starting at $\tau$ like that defined in lemma 2, then we have $\|d\mathbf{v}_\tau\| \le K_1 \|\mathbf{x}^\tau\|$ for some bound $K_1$ independent of $\tau$.

Further, picking any $\gamma$ such that $\beta < \gamma < 1$, we have some bound $K_2 > 0$ such that $\|d\mathbf{v}_t\| \le K_2 \gamma^{\tau - t} \|d\mathbf{v}_\tau\|$.[A-9] The differentiability of $\Lambda$ and $c$ further implies that $\|d\Lambda_t\| \le K_3 \|d\mathbf{v}_{t+1}\|$ and $\|d\mathbf{c}_t\| \le K_4 \|d\mathbf{v}_{t+1}\|$ for some bounds $K_3, K_4 > 0$ (assuming that $t < \tau$). Combining all these bounds, we have $\|d\Lambda_t\| \le K_5 \gamma^{\tau - t} \|\mathbf{x}^\tau\|$ and $\|d\mathbf{c}_t\| \le K_6 \gamma^{\tau - t} \|\mathbf{x}^\tau\|$ for $K_5 = K_1 K_2 K_3 / \gamma$ and $K_6 = K_1 K_2 K_4 / \gamma$.

*Step 3.*

Iterating equation (A.16), we have that

$$d\mathbf{D}_t = \sum_{s=0}^{t-1} (\Lambda^{t-s-1}) d\Lambda_s$$

[A-7] For instance, if we discretize the value function and the distribution on a grid with $N$ points, $\mathbf{v}_t$, $\mathbf{D}_t$, $v(\mathbf{v}_{t+1}, Z_t)$ and $c(\mathbf{v}_{t+1}, Z_t)$ are $N \times 1$ and $\Lambda(\mathbf{v}_{t+1}, Z_t)$ is an $N \times N$ transition matrix.

[A-8] To show injectivity of $(d\mathbf{v}_0, d\mathbf{v}_1, \ldots) \to (d\mathbf{v}_0 - \mathbf{v_v} d\mathbf{v}_1, d\mathbf{v}_1 - \mathbf{v_v} d\mathbf{v}_2, \ldots)$, note that if the output is zero, then we have $d\mathbf{v}_t = (\mathbf{v_v})^s d\mathbf{v}_{t+s}$ for any $t, s > 0$. Therefore $\|d\mathbf{v}_t\| \le \|\mathbf{v}_\mathbf{v}^s\| \|d\mathbf{v}_{t+s}\| \le \|\mathbf{v}_\mathbf{v}^s\| \|d\mathbf{v}\|$, and taking the limit as $s \to 0$ the right side of this inequality becomes 0 (because $(\mathbf{v_v})^s \to 0$, as follows from spectral radius $\rho(\mathbf{v_v}) < 1$), so that $d\mathbf{v}$ must be zero as well. To show surjectivity, given a desired output $(d\mathbf{w}_0, d\mathbf{w}_1, \ldots)$ we can construct $d\mathbf{v}_t = \sum_{s=0}^\infty (\mathbf{v_v})^s d\mathbf{w}_{t+s}$, where we note similarly that $\|d\mathbf{v}\| \le \|d\mathbf{w}\| \sum_{s=0}^\infty \|\mathbf{v}_\mathbf{v}^s\|$, where the infinite sum converges because $\rho(\mathbf{v_v}) < 1$.

[A-9] To see this, note that $\|d\mathbf{v}_t\| \le \|\mathbf{v}_\mathbf{v}^{\tau - t}\| \|d\mathbf{v}_\tau\|$, so that $\|d\mathbf{v}_t / \delta^{\tau - t}\| \le \|(\mathbf{v_v}/\delta)^{\tau - t}\| \|d\mathbf{v}_\tau\|$, and that $(\mathbf{v_v}/\delta)^{\tau - t} \to 0$ as $\tau \to \infty$ since its spectral radius $\beta/\delta$ is less than 1.

where $\Lambda$ is the steady-state transition matrix. Since the spectral radius of $\Lambda$ is at most 1, if we pick any $\lambda$ such that $1 < \lambda < \gamma^{-1}$, analogously to above we have $\|\Lambda^{t-s-1}\| \le K_7\lambda^{t-s-1}$ for some $K_7 > 0$. Combining this with our previous bound, we have

$$\begin{aligned}
\|d\mathbf{D}_t\| &\le \sum_{s=0}^{t-1} \|\Lambda^{t-s-1}\| \|d\Lambda_s\| \\
&\le \|\mathbf{x}^\tau\| \sum_{s=0}^{t-1} K_7\lambda^{t-s-1} K_5 \gamma^{\tau-s} = \|\mathbf{x}^\tau\| K_5 K_7 \sum_{s=0}^{t-1} (\lambda\gamma)^{t-s}\gamma^{\tau-t} \\
&< \frac{K_5 K_7}{1-\delta\lambda}\gamma^{\tau-t}\|\mathbf{x}^\tau\| \equiv K_8\gamma^{\tau-t}\|\mathbf{x}^\tau\|
\end{aligned}$$

*Step 4.*

Finally (letting $\mathbf{D}$ and $\mathbf{c}$ denote the steady-state distribution and individual consumption):

$$\begin{aligned}
|dC_t| &\le \|d\mathbf{c}_t\|\|\mathbf{D}\| + \|d\mathbf{D}_t\|\|\mathbf{c}\| \\
&\le (K_6\|\mathbf{D}\| + K_8\|c\|)\gamma^{\tau-t}\|\mathbf{x}^\tau\|
\end{aligned}$$

which clearly satisfies condition (A.14) with the arbitrary $\gamma \in (\beta, 1)$ chosen above and $K \equiv K_6\|\mathbf{D}\| + K_8\|c\|$. □

We verify numerically that all of our quantitative models used in the paper satisfy the two conditions of proposition 9, and therefore that they have a matrix representation.

**Counterexample: when does an operator on $\ell^\infty$ not have an infinite matrix representation?** Now that we have demonstrated the existence of an infinite matrix representation of $\mathbf{M}$ under mild conditions on our economic problem, it is worth asking what a counterexample might look like.

Consider a operator $\mathbf{M} : c \to c$, where $c \subset \ell^\infty$ is the space of *convergent* sequences, endowed with the sup norm. Suppose that this is defined for any $\mathbf{x} \in c$ by

$$\begin{aligned}
(\mathbf{Mx})_0 &\equiv \lim_{t\to\infty} x_t \\
(\mathbf{Mx})_1 &\equiv (1+r)\left(\mathbf{q}'\mathbf{x} - (\mathbf{Mx})_0\right) \\
(\mathbf{Mx})_t &\equiv 0 \qquad \forall t > 1
\end{aligned}$$

Observe that this $\mathbf{M}$ satisfies our present value condition $\mathbf{q}'\mathbf{M} = \mathbf{q}'$.

Any infinite matrix $[M_{ts}]_{t,s=0}^\infty$ representing $\mathbf{M}$ would need to have all zeros in the 0th row, because we have $(\mathbf{Me}_s)_0 = 0$ for any $s$. But then for, for instance, the sequence of all ones $\mathbf{x} = (1, 1, 1, \ldots)$, we have $(\mathbf{Mx})_0 = 1$, which is inconsistent with multiplication by a row of all zeros. We conclude that an infinite matrix representation $[M_{ts}]_{t,s=0}^\infty$ of this operator, in the sense of definition 1, is impossible.[A-10] Although this counterexample is for $c$ rather than $\ell^\infty$, it can be extended to $\ell^\infty$

[A-10] If an additional row and column are added to the infinite matrix to represent $(1, 1, \ldots)$, then an infinite matrix using the Hahn-Banach theorem.[A-11]

This counterexample illustrates what it might mean to violate (A.14), such that an infinite matrix representation is not possible: the indefinitely far future—in this case literally the *limit* of income at infinity—must matter for present consumption. This seems implausible for any economic example, especially when $r > 0$, so that the present value of income in the infinite future is zero.

### A.5 Proof of proposition 1

We are now ready to prove proposition 1.

*Proof of proposition 1.* Given any $\{G_t, T_t\}$ satisfying the intertemporal budget constraint $\sum_{t=0}^{\infty} \frac{G_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{T_t}{(1+r)^t}$, the path of output must satisfy (11). Consider a bounded shock $d\mathbf{G}$, $d\mathbf{T}$ satisfying $\mathbf{q}' d\mathbf{G} = \mathbf{q}' d\mathbf{T}$. Differentiating (11), we find that the impulse response of output $d\mathbf{Y} = \{dY_t\}$ must satisfy

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial Z_s} \left(dY_s - dT_s\right) = dG_t - \sum_{s=0}^{\infty} M_{ts} dT_s + \sum_{s=0}^{\infty} M_{ts} dY_s \tag{A.18}$$

and (13) follows by stacking (A.18).

Starting from the aggregated budget constraint (A.5), and applying repeated substitution, using that $A_t\left(\{Z_s\}\right) = A_{-1}$ is a predetermined variable and that the boundedness of $\mathcal{A}_t$ implies $\lim_{t\to\infty} \frac{A_t(\{Z_s\})}{(1+r)^t} = 0$, we obtain:

$$\sum_{t=0}^{\infty} \frac{\mathcal{C}_t\left(\{Z_s\}\right)}{(1+r)^t} = (1+r)\, A_{-1} + \sum_{t=0}^{\infty} \frac{Z_t}{(1+r)^t}$$

Taking derivatives of this equation respect to $Z_s$, we arrive at:

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} M_{ts} = \frac{1}{(1+r)^s} \quad \forall s$$

□

### A.6 Proof of proposition 2

On any sequence space $\ell^p$, we define the lag operator $\mathbf{L}$ as the operator $\{x_0, x_1, \ldots\} \to \{0, x_0 \ldots\}$, and the lead (or forward) operator $\mathbf{F}$ as the operator $\{x_0, x_1, \ldots\} \to \{x_1, x_2 \ldots\}$. Note that these

representation is possible, but this is not the notion of infinite matrix we use in definition 1, where the $s$th row or column is always $\mathbf{e}_s$. This reflects the fact when $(1, 1, \ldots)$ is added to the usual $\mathbf{e}_s$, they form a Schauder basis for $c$. $\ell^\infty$ does not have a Schauder basis, and one can construct even more complex counterexamples that are specific to $\ell^\infty$.

[A-11] In more detail: treating $(\mathbf{Mx})_0$ as a linear functional on $\mathbf{x} \in c$, and noting that $(\mathbf{Mx})_0 \le \|\mathbf{x}\|$, we apply the Hahn-Banach theorem to extend it to a linear functional on $\mathbf{x} \in \ell^\infty$, which continues to satisfy $(\mathbf{Mx})_0 \le \|\mathbf{x}\|$ and coincides with the original functional on $c$. We then define $(\mathbf{Mx})_1$ as before, and the same argument goes through: the infinite matrix needs to have all zeros in the 0th row, but that gives the wrong answer for $\mathbf{x} = (1, 1, 1, \ldots)$.

satisfy

$$\mathbf{FL} = \mathbf{I} \quad \text{and} \quad \mathbf{LF} = \mathbf{I} - \mathbf{e}_0\mathbf{e}_0' \neq \mathbf{I}$$

where $\mathbf{e}_0$ is the vector with all entries equal to 0 except the first entry equal to 1, so that the first row and first column of $\mathbf{LF}$ are both zero. Hence, $\mathbf{F}$ is a left but not a right inverse of $\mathbf{L}$. Moreover, $\mathbf{L}$ is injective but not surjective; while $\mathbf{F}$ is surjective but not injective.

**Lemma 3.** *The asset Jacobian* $\mathbf{A}$*, with elements* $A_{ts} = \frac{\partial \mathcal{A}_t}{\partial Z_s}$*, and the consumption Jacobian* $\mathbf{M}$*, with elements* $M_{ts} = \frac{\partial \mathcal{C}_t}{\partial Z_s}$*, are related by the formula:*

$$\mathbf{I} - \mathbf{M} = (\mathbf{I} - (1+r)\mathbf{L})\mathbf{A} \tag{A.19}$$

*Proof.* Differentiating (A.5) at $t$ with respect to any $Z_s$, we obtain

$$M_{ts} + A_{ts} = (1+r)\,A_{t-1,s} + 1_{t=s}$$

stacking these terms and using the definition of $\mathbf{L}$, we obtain (A.19). □

**Lemma 4.** *The operator* $\mathbf{K}$*, defined as* $\mathbf{K} \equiv -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^t$ *(see proposition 2), satisfies:*

$$\mathbf{K}(\mathbf{I} - (1+r)\mathbf{L}) = \mathbf{I} \qquad \text{and} \qquad (\mathbf{I} - (1+r)\mathbf{L})\mathbf{K} = \mathbf{I} - \mathbf{e}_0\mathbf{q}'$$

*Proof.* Since $\mathbf{FL} = \mathbf{I}$, we have:

$$\begin{aligned}
\mathbf{KL} &= -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^t\mathbf{L} \\
&= -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^{t-1} \\
&= -(1+r)^{-1}\sum_{t=0}^{\infty}(1+r)^{-t}\mathbf{F}^t \\
&= (1+r)^{-1}(\mathbf{K} - \mathbf{I})
\end{aligned}$$

This implies that $\mathbf{K}(\mathbf{I} - (1+r)\mathbf{L}) = \mathbf{K} - (1+r)\mathbf{KL} = \mathbf{K} - (\mathbf{K} - \mathbf{I}) = \mathbf{I}$. Similarly, since $\mathbf{LF} = \mathbf{I} - \mathbf{e}_0\mathbf{e}_0'$ we have:

$$\begin{aligned}
\mathbf{LK} &= -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{LF}^t \\
&= \left(\mathbf{I} - \mathbf{e}_0\mathbf{e}_0'\right)(1+r)^{-1}(\mathbf{K} - \mathbf{I})
\end{aligned}$$

so that $(\mathbf{I} - (1+r)\mathbf{L})\mathbf{K} = \mathbf{K} - \left(\mathbf{I} - \mathbf{e}_0\mathbf{e}_0'\right)(\mathbf{K} - \mathbf{I}) = \mathbf{I} - \mathbf{e}_0\mathbf{e}_0'(\mathbf{I} - \mathbf{K}) = \mathbf{I} - \mathbf{e}_0\mathbf{q}'$, where the last equality follows from the fact that the first row of $\mathbf{I} - \mathbf{K}$ is equal to $\mathbf{q}'$. □

We can now turn to the proof of the proposition.

*Proof of proposition 2.* Lemmas 3 and 4 together imply that, with the choice of $\mathbf{K}$ in the proposition,

$$\mathbf{K}(\mathbf{I}-\mathbf{M}) = \mathbf{K}(\mathbf{I}-(1+r)\mathbf{L})\mathbf{A} = \mathbf{A}$$

Suppose first that $\mathbf{A}$ is invertible. We can write the IKC (13) as $(\mathbf{I}-\mathbf{M})d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T}$. Multiplying by $\mathbf{K}$ on both sides, we obtain

$$\mathbf{A}d\mathbf{Y} = \mathbf{K}(d\mathbf{G} - \mathbf{M}d\mathbf{T}) \tag{A.20}$$

This has a unique solution $d\mathbf{Y} = \mathbf{A}^{-1}\mathbf{K}(d\mathbf{G} - \mathbf{M}d\mathbf{T})$, so this is the unique solution to the IKC for any $d\mathbf{G}, d\mathbf{T} \in \ell^\infty$. In this case, we define $\mathcal{M} \equiv \mathbf{A}^{-1}\mathbf{K}$. This is obviously a linear operator, and is bounded since $\mathbf{A}^{-1}$ is bounded by the bounded inverse theorem, and $\mathbf{K}$ is bounded.

Now suppose that $\mathbf{A}$ is not invertible. By the bounded inverse theorem, this can only happen if it fails to be injective or surjective.

Suppose $\mathbf{A}$ is not injective, i.e. $\mathbf{A}\mathbf{v} = 0$ for some $\mathbf{v}$. Since $\mathbf{I}-\mathbf{M} = (\mathbf{I}-(1+r)\mathbf{L})\mathbf{A}$, it immediately follows that $(\mathbf{I}-\mathbf{M})\mathbf{v} = 0$, so that if $d\mathbf{Y}$ is a solution to the IKC, then $d\mathbf{Y} + \mathbf{v}d\lambda$ is also a solution for any $d\lambda \in \mathbb{R}$. Hence, if $\mathbf{A}$ is not injective, any solution to the IKC is non unique.

Suppose instead that $\mathbf{A}$ is not surjective. This implies that there is some $\mathbf{x}$ such that we cannot find any $\mathbf{v}$ such that $\mathbf{x} = \mathbf{A}\mathbf{v}$. Hence, there is no $\mathbf{v}$ such that $\mathbf{x} = \mathbf{K}(\mathbf{I}-\mathbf{M})\mathbf{v}$. Now, $\mathbf{K}$ is Toeplitz and has winding number of $-1$, which means that it has a one-dimensional kernel and is surjective.[A-12] This kernel is spanned by the vector $\{1, 0, 0, \ldots\}$. Since $\mathbf{K}$ is surjective, there is some $\mathbf{y}$ such that $\mathbf{x} = \mathbf{K}\mathbf{y}$, and by adding and subtracting $\{1, 0, 0, \ldots\}$ we can choose $\mathbf{y}$ so that it satisfies $\mathbf{q}'\mathbf{y} = \mathbf{0}$. Let $d\mathbf{G} = \mathbf{y}$. This shows that there is no $\mathbf{v}$ such that $d\mathbf{G} = (\mathbf{I}-\mathbf{M})\mathbf{v}$, and therefore that, for this shock satisfying $\mathbf{q}'d\mathbf{G} = 0$, the IKC does not have a solution. Hence, if $\mathbf{A}$ is not surjective, $\mathbf{I}-\mathbf{M}$ is not surjective on the space of zero-NPV vectors, in other words, there exists shocks such that the IKC does not have a solution.

This completes the proof that there exists a unique solution to the IKC for any shock if, and only if, $\mathbf{A} = \mathbf{K}(\mathbf{I}-\mathbf{M})$ is invertible.

In the case where $\mathbf{A}$ is invertible, we further have:

$$\mathcal{M}(\mathbf{I}-\mathbf{M}) = \mathbf{A}^{-1}\mathbf{K}(\mathbf{I}-(1+r)\mathbf{L})\mathbf{A} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

where we have used the definition of $\mathcal{M}$, lemma 3 and lemma 4. Hence, $\mathcal{M}$ is a left inverse of

[A-12] Böttcher and Grudsky (2005), Theorem 1.9, shows that the Fredholm index equals minus the winding number in any $\ell^p, 1 \leq p \leq \infty$. This implies that the difference between the dimension of the kernel and cokernel of $\mathbf{K}$ is 1. Since it is immediate that $\{1, 0, 0, \ldots\}$ is the only element of the kernel of $\mathbf{K}$, this means that $\mathbf{K}$ has cokernel dimension zero, so that $\mathbf{K}$ is surjective.

$\mathbf{I}-\mathbf{M}$ on $\ell^\infty$. Moreover, we have:

$$\begin{aligned}(\mathbf{I}-\mathbf{M})\,\mathcal{M} &= (\mathbf{I}-(1+r)\mathbf{L})\mathbf{A}\mathbf{A}^{-1}\mathbf{K}\\ &= (\mathbf{I}-(1+r)\mathbf{L})\mathbf{K}\\ &= \mathbf{I}-\mathbf{e}_0\mathbf{q}'\end{aligned}$$

so, in particular, for any $\mathbf{x}$ such that $\mathbf{q}'\mathbf{x}=0$, we have $(\mathbf{I}-\mathbf{M})\,\mathcal{M}\mathbf{x}=\mathbf{x}$. □

### A.7 Numerically solving the IKC

In practice, on the computer, we calculate Jacobians by following the method in Auclert et al. (2021a), truncating to a horizon of $T$, so that $\mathbf{M}$ is a $T\times T$ matrix, and $d\mathbf{G}$, $d\mathbf{T}$ are $T\times 1$ vectors. Truncating $\mathbf{M}$ generally implies that $\mathbf{q}'(\mathbf{I}-\mathbf{M})\neq 0$, since the "tents" corresponding to the final columns of $\mathbf{M}$ (see figure 3) are incomplete.

A first approach is to directly solve for the multiplier matrix $\mathcal{M}$, numerically computing $\mathcal{M}=\mathbf{A}^{-1}\mathbf{K}$, where $\mathbf{A}$ is the asset Jacobian, whose elements are given by $A_{ts}=\frac{\partial\mathcal{A}_t}{\partial Z_s}$. We obtain $\mathbf{A}$ either directly using the methods from Auclert et al. (2021a), or indirectly from $\mathbf{M}$ using $\mathbf{A}=\mathbf{K}(\mathbf{I}-\mathbf{M})$.[A-13] Then, given $\mathcal{M}$, we form

$$d\mathbf{Y}=\mathcal{M}\,(d\mathbf{G}-\mathbf{M}d\mathbf{T}) \tag{A.21}$$

Second, we can solve the model in the asset space. Recall from the proof of proposition 2 that any solution to the IKC (13) must also solve

$$\mathbf{A}\,(d\mathbf{Y}-d\mathbf{T})=d\mathbf{B} \tag{A.22}$$

We obtain $d\mathbf{B}$ from the present-value budget constraint, using $d\mathbf{B}=\mathbf{K}\,(d\mathbf{G}-d\mathbf{T})$. We then solve the matrix equation (A.22) numerically.

Note that this is closely related to the first approach, since (A.22) gives

$$d\mathbf{Y}=\mathbf{A}^{-1}d\mathbf{B}+d\mathbf{T}=\mathbf{A}^{-1}\mathbf{K}\,(d\mathbf{G}-d\mathbf{T})+d\mathbf{T}=\mathcal{M}\,(d\mathbf{G}-d\mathbf{T})+d\mathbf{T}$$

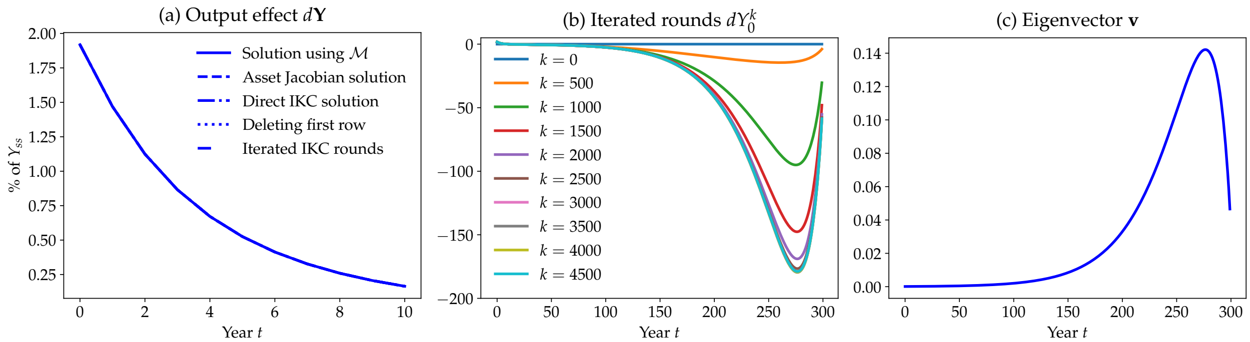
while (A.21) delivers

$$d\mathbf{Y}=\mathcal{M}\,(d\mathbf{G}-\mathbf{M}d\mathbf{T})=\mathcal{M}\,(d\mathbf{G}-d\mathbf{T})+\mathcal{M}\,(\mathbf{I}-\mathbf{M})\,d\mathbf{T}$$

and we can check that $\mathcal{M}\,(\mathbf{I}-\mathbf{M})=\mathbf{I}$ holds numerically except for truncation error.

[A-13] Calculating a $T\times T$ matrix $\mathbf{A}$ in this way requires first calculating $\overline{T}\times\overline{T}$ matrices $\mathbf{K}$ and $\mathbf{I}-\mathbf{M}$, with $\overline{T}$ sufficiently larger than $T$, and then keeping the first $T\times T$ entries. Otherwise, truncation error appears. For instance, since the last row of $\mathbf{K}$ is zero, the last row of $\mathbf{A}$ is also zero, making $\mathbf{A}$ non-invertible. This is closely related to the numerical problems that we find when solving the IKC in the goods space.

Figure A.1: Five ways of solving the IKC numerically




A third approach is to instead solve numerically the goods-space matrix equation

$$(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T} \tag{A.23}$$

which is the rearranged IKC. Recall that generally, with truncation, we have $\mathbf{q}'(\mathbf{I}-\mathbf{M}) \neq 0$, so $\mathbf{I}-\mathbf{M}$ is numerically invertible. For large $T$, however, this becomes numerically close to degenerate, and this goods-space approach is subject to greater numerical error than the asset-space approach.

A fourth approach is similar to the first, directly obtaining $\mathcal{M} = (\mathbf{K}(\mathbf{I}-\mathbf{M}))^{-1}\mathbf{K}$ and then $d\mathbf{Y} = \mathcal{M}(d\mathbf{G} - \mathbf{M}d\mathbf{Y})$, but replaces the $\mathbf{K}$ we defined earlier with the forward operator $\mathbf{F}$. In practice, this approach can be implemented by deleting the first row and last column of $\mathbf{I}-\mathbf{M}$, inverting it, and then applying this inverse to $d\mathbf{G} - \mathbf{M}d\mathbf{Y}$ with the first entry deleted.

This approach delivers similar accuracy to the first two approaches and, in the absence of an asset Jacobian $\mathbf{A}$, can be slightly easier to implement. Intuitively, the approach works because the IKC has a redundancy: assuming that $\mathbf{q}'d\mathbf{G} = \mathbf{q}'d\mathbf{T}$, the present value of both sides is always equal, for any $d\mathbf{Y}$. To deal with this redundancy, we drop a single equation. Mathematically, this approach can be motivated for stationary models by Auclert et al. (2023b): the forward operator $\mathbf{F}$ has the same winding number, -1, as the $\mathbf{K}$ we defined earlier, and composing it with $\mathbf{I}-\mathbf{M}$ (which has winding number 1) again gives an invertible operator with winding number 0.

Finally, a fifth approach is to implement an analog of the iterative solution (7) to the static Keynesian cross. To do this, we first form the sequence $d\mathbf{Y}^{(k)}$

$$d\mathbf{Y}^{(k)} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}^{(k-1)} \tag{A.24}$$

by repeated iterations. This sequence converges numerically to a certain $d\mathbf{Y}^{\infty} = \sum_{s=0}^{\infty} \mathbf{M}^s(d\mathbf{G} - \mathbf{M}d\mathbf{T})$. For some of the applications in this paper, such as our calibrated BU and TABU models, $d\mathbf{Y}^{\infty}$ equal to the unique bounded solution $d\mathbf{Y}$, which we obtain via one of the methods discussed above. For other applications, such as our calibrated HA-one model, however, $d\mathbf{Y}^{\infty}$ differs from $d\mathbf{Y}$; instead, it is explosive. Interestingly, however, we find that $d\mathbf{Y}^{\infty}$ differs from $d\mathbf{Y}$ by a constant multiple of the the eigenvector $\mathbf{v}$ corresponding to the highest eigenvalue of $\mathbf{M}$; that is, we have

$d\mathbf{Y} = d\mathbf{Y}^\infty + d\lambda\mathbf{v}$. In practice, therefore, to obtain $d\mathbf{Y}$ from $d\mathbf{Y}^\infty$, we can find the eigenvalue $\mathbf{v}$ and look for the unique $d\lambda$ such that $dY_s = 0$ for some sufficiently large $s$.[A-14]

For our one-account HA model solved with a sequence of government spending $dG_t = 0.8^t$ and $dB_t = 0.5 \cdot (dB_{t-1} + dG_t)$, figure A.1(a) shows that these four alternatives deliver the same solution, up to some small error. Figure A.1(b) shows the outcome of the iteration process in equation (A.24), and figure A.1(c) displays the leading eigenvector $\mathbf{v}$ of the $\mathbf{M}$ matrix. The sequence $d\mathbf{Y}_0^k$ converges slowly and to a limit very different from the true solution $d\mathbf{Y}$, but we obtain the right solution after correcting for a multiple of $\mathbf{v}$.

### A.8 Alternative allocation rules and other shocks

Consider now the case with a general allocation rule, $n_{it} = \mathcal{N}(e_{it}, N_t)$, subject to $\int \mathcal{N}(e_{it}, N_t)di = N_t$, a general retention function $z_{it} = \mathcal{Z}_t(e_{it}n_{it}, T_t, Y_t)$, time-varying real interest rates $r_t$, and a general shifter $\Theta$ to the household consumption problem. Production is still linear in effective labor, so $Y_t = N_t$, and prices are still flexible, so the real wage is $w_t = 1$. Hence, total taxes $T_t = w_t N_t - \int z_{it} di$ are now given by

$$T_t = Y_t - \int \mathcal{Z}_t\left(e_{it}\mathcal{N}(e_{it}, Y_t), T_t, Y_t\right) di$$

This equation implicitly defines $\mathcal{Z}_t(\cdot, T_t, Y_t)$ as a function of $Y_t$ and $T_t$. Hence, the aggregates that matter for household devisions are now $\mathbf{X}_t = \{Y_t, T_t, r_t, \Theta\}$ as given when making their decisions. Applying the result from section A.1, the consumption function is now:

$$C_t = \mathcal{C}_t\left(\{Y_s, T_s, r_s, \Theta\}\right) \tag{A.25}$$

and there is also an aggregate asset function $\mathcal{A}_t(\{Y_s, T_s, r_s, \Theta\})$, related to $\mathcal{C}_t$ via

$$\mathcal{C}_t\left(\{Y_s, T_s, r_s, \Theta\}\right) + \mathcal{A}_t\left(\{Y_s, T_s, r_s, \Theta\}\right) = (1 + r_{t-1})\,\mathcal{A}_{t-1}\left(\{Y_s, T_s, r_s, \Theta\}\right) + Y_t - T_t \tag{A.26}$$

Any path $\{Y_t\}$ that is part of an equilibrium must satisfy

$$\mathcal{C}_t\left(\{Y_s, T_s, r_s, \Theta\}\right) + G_t = Y_t$$

[A-14] Intuitively, provided that $\mathbf{M} > 0$, which is the case in our applications, then the operator $\tilde{\mathbf{M}}$ defined as $\tilde{M}_{ts} = \left(\frac{1}{1+r}\right)^{t-s} M_{ts}$ is a Markov chain on the natural numbers $\mathbb{N}$. Then $\tilde{\mathbf{M}}$ can either be transient or recurrent. When $\tilde{\mathbf{M}}$ is transient, as in our calibrated TABU model, the Markov chain does not admit a stationary measure and the IKC rounds converge. When $\tilde{\mathbf{M}}$ is recurrent, as in our HA-one model, the Markov chain admits a stationary measure $\tilde{\mathbf{v}}$, which is such that $\tilde{\mathbf{M}}\tilde{\mathbf{v}} = \tilde{\mathbf{v}}$, ie $\tilde{v}_t = \sum_{s=0}^{\infty} \tilde{M}_{ts}\tilde{v}_s = \left(\frac{1}{1+r}\right)^{t-s} M_{ts}\tilde{v}_s$. Then, we have that the sequence $\mathbf{v}$ defined by $v_t \equiv (1+r)^t\,\tilde{v}_t$ satisfies $\mathbf{M}\mathbf{v} = \mathbf{v}$: this looks like a vector of self-sustaining demand, but it is unbounded. When we truncate $\mathbf{M}$, we find that the largest eigenvalue is close to but not exactly equal to 1, and that the eigenvector, satisfying $\mathbf{M}\mathbf{v} \simeq \mathbf{v}$, is close to the $\mathbf{v}$ from the recurrent chain. Since there is a unique bounded solution to the IKC, it must differ from the one we obtain from iterated rounds $d\mathbf{Y}^\infty$ by some multiple of $\mathbf{v}$ that is just enough to keep $d\mathbf{Y}$ bounded.

which is equation (17), the new non-linear fixed point equation for output, which we seek to solve for any path $\{G_t, T_t, r_t\}$ that satisfies the government budget constraint,

$$\sum_{t=0}^{\infty} \frac{T_t}{\Pi_{s=0}^{t-1}(1+r_s)} = \sum_{t=0}^{\infty} \frac{G_t}{\Pi_{s=0}^{t-1}(1+r_s)} + (1+r_{-1})\, B_{-1} \tag{A.27}$$

Consider a perturbation to $\{d\mathbf{G}, d\mathbf{T}, d\mathbf{r}\}$ that respects (A.27), starting from the steady state (recall that we define the vector $d\mathbf{r}$ to have elements $d\log(1+r_t) = \frac{dr_t}{1+r}$). Totally differentiating (A.27), we have

$$\sum_{t=0}^{\infty} \frac{dT_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{dr_t}{1+r} \sum_{s=t}^{\infty} \frac{T}{(1+r)^s} = \sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{dr_t}{1+r} \sum_{s=t}^{\infty} \frac{G}{(1+r)^s}$$

and since $B = \sum_{t=0}^{\infty} \frac{T-G}{(1+r)^t}$, is also

$$\sum_{t=0}^{\infty} \frac{dT_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t} + B \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \frac{dr_t}{1+r}$$

or in vector notation, $\mathbf{q}' d\mathbf{T} = \mathbf{q}' d\mathbf{G} + B\mathbf{q}' d\mathbf{r}$. Then, totally differentiating (17) we have:

$$\sum_{t=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial Y_s} dY_s + \sum_{t=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial T_s} dT_s + \sum_{t=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial \log(1+r_s)} \frac{dr_s}{1+r} + \frac{\partial \mathcal{C}_t}{\partial \Theta} d\Theta + dG_t = dY_t$$

and writing $M_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial Y_s}$, $M^T_{t,s} \equiv -\frac{\partial \mathcal{C}_t}{\partial T_s}$, $M^r_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial \log(1+r_s)}$and $\partial C_t \equiv \frac{\partial \mathcal{C}_t}{\partial \Theta} d\Theta$, this is

$$\sum_{t=0}^{\infty} M_{ts} dY_s - \sum_{t=0}^{\infty} M^T_{t,s} dT_s + \sum_{t=0}^{\infty} M^r_{t,s} \frac{dr_s}{1+r} + \partial C_t + dG_t = dY_t$$

which now, in stacked form, is equation (18).

Next, integrating (A.26), we have that

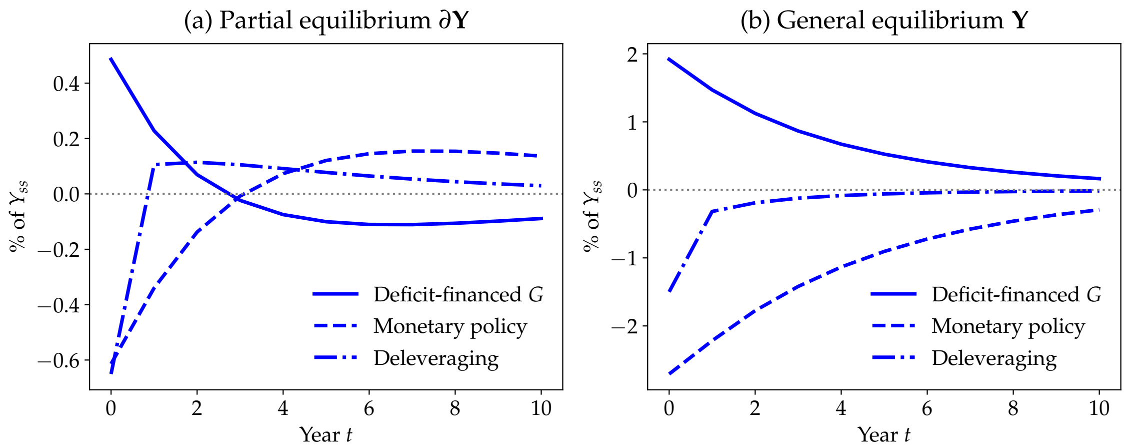
$$\sum_{t=0}^{\infty} \frac{\mathcal{C}_t(\{Y_s, T_s, r_s, \Theta\})}{\Pi_{s=0}^{t-1}(1+r_s)} = (1+r)\, A_{-1} + \sum_{t=0}^{\infty} \frac{Y_t - T_t}{\Pi_{s=0}^{t-1}(1+r_s)}$$

Totally differentiating with respect to $Y_s$ and $T_s$, we see that $\mathbf{q}'\mathbf{M} = \mathbf{q}'\mathbf{M}^T = \mathbf{q}'$. Totally differentiating with respect to $\Theta$, we see that $\mathbf{q}'\partial\mathbf{C} = 0$, that is, shifters to consumption reshuffle the path of consumption without altering its present value. Finally, totally differentiating with respect to $\log(1+r_s)$, we see that

$$\sum_{t=0}^{\infty} \frac{M^r_{ts}}{(1+r)^t} \frac{dr_s}{1+r} = -\frac{dr_s}{1+r} \sum_{u=s}^{\infty} \frac{Y-T-C}{(1+r)^s} = A \frac{1}{(1+r)^s} \frac{dr_s}{1+r}$$

which shows that $\mathbf{q}'\mathbf{M}^r = A\mathbf{q}'$, that is, the income effect of a change in the real interest rate must

Figure A.2: Partial and general equilibrium impulses




Note: The figure shows, in our calibrated HA-one model, the partial equilibrium effect $\partial\mathbf{Y}$ and the general equilibrium effect $d\mathbf{Y} = \mathcal{M}d\mathbf{Y}$ of three shocks. The deficit-financed government spending shock $d\mathbf{G}, d\mathbf{T}$, has $dG_t = 0.8^t dG_0$, $dG_0$ equal to 1% of GDP, and gives rise to a path of debt $dB_t = 0.5^t dG_0$. The monetary policy shock $d\mathbf{r}$ has $dr_t = 0.8^t$, with a government running a balanced budget and adjusting taxes at the margin in response to changes in interest rate expenses. The deleveraging shock is a shock $d\underline{a} = 0.8^t$ to the minimal assets of agents.

aggregate, in present value terms, to the value of household assets. Taken together, we have that

$$\mathbf{q}'\left(\mathbf{M}^r d\mathbf{r} + \partial\mathbf{C} + d\mathbf{G} - \mathbf{M}^T d\mathbf{T}\right) = \mathbf{q}'\mathbf{M}^r d\mathbf{r} + 0 + \mathbf{q}' d\mathbf{G} - \mathbf{q}' d\mathbf{T} = A\mathbf{q}' d\mathbf{r} - B\mathbf{q}' d\mathbf{r} = 0$$

That is, the partial equilibrium impulse to spending has zero present value, including for shocks to real interest rates, since the positive income effects to the household sector are offset by negative income effects to the government sector that require an increase in taxes. Hence, (18) has exactly the same form as (13), and can be solved similarly if and only if the asset Jacobian $\mathbf{A}$ with elements $A_{ts} = \frac{\partial \mathcal{A}_t}{\partial Y_s}$ is invertible, in which case, defining $\mathcal{M} = \mathbf{A}^{-1}\mathbf{K}$, the unique solution is given by (19).

**Applications: fiscal policy, monetary policy and deleveraging.** We use our calibrated HA-one model to illustrate these concepts in figure A.2. We consider three shocks: a deficit-financed $G$ shock, a monetary policy shock, and a deleveraging shock. We provide a fourth application, to lump-sum taxation, in appendix E.1.

Our first shock is a deficit-financed government spending shock $d\mathbf{G}, d\mathbf{T}$, with $dG_t = 0.8^t dG_0$, $dG_0$ equal to 1% of GDP, and that gives rise to a path of debt $dB_t = 0.5^t dG_0$. The partial equilibrium impulse, $\partial\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T}$, is displayed on the solid line in panel (a), and clearly has present value 0. Initially, partial equilibrium output is boosted by government spending, and later, it is depressed by the lower consumption due to higher taxes. The general equilibrium output response is displayed in panel (b), and we can verify that $d\mathbf{Y} = \mathcal{M}\partial\mathbf{Y}$. Note that this is positive everywhere, and much larger in magnitude than the impact $\partial\mathbf{Y}$.

Our second shock is a monetary policy shock $d\mathbf{r}$, with $\frac{dr_t}{1+r} = 0.8^t$. We assume that the government runs a balanced budget, with all changes in interest rate expenses resulting in a contemporaneous tax adjustment. Let $\mathbf{J}^{T,r}$ be the resulting Jacobian of taxes to interest rates: then, the partial equilibrium impulse is $\partial \mathbf{Y} = \left(\mathbf{M}^r - \mathbf{M}\mathbf{J}^{T,r}\right) d\mathbf{r}$, which has zero present value, $\mathbf{q}'\partial \mathbf{Y} = 0$ and is displayed in the dashed line of panel (a). This is a "contractionary" shock, so it lowers output initially, but in partial equilibrium it also raises it later on. In general equilibrium, again, after adjustment of incomes, we obtain a contraction of output at every date, as panel (b) shows. Since the monetary $\partial \mathbf{Y}$ is roughly flipped and double the size of the fiscal $\partial \mathbf{Y}$ in this example, the general equilibrium monetary $d\mathbf{Y}$ is also roughly flipped and double the size of the fiscal $d\mathbf{Y}$. This makes sense since it is the same $\mathcal{M}$ that translates partial to general equilibrium outcomes in both cases.

Our third shock is a deleveraging shock, $d\underline{a} = 0.8^t$, which tightens the borrowing constraint and then progressively relaxes it.[A-15] As the dash-dotted lines in panel (c) show, this leads to an immediate contraction in partial equilibrium spending followed by an expansion later on, as agents are forced to save and can then spend down their excess savings as the constraint is relaxed. Again, the shock has a zero NPV effect on partial equilibrium output, $\mathbf{q}'\partial \mathbf{Y} = 0$, but in general equilibrium, after adjustment of incomes, this shock leads to a recession that is not followed by a boom. Here also, we can see the same multiplier $\mathcal{M}$ translating partial equilibrium impulses to general equilibrium outcomes, amplifying the initial shock.

# B Beyond the intertemporal Keynesian cross

This section shows how additional macroeconomic channels can be incorporated into the analysis while maintaining a generalized intertemporal Keynesian cross that still has the form in (18), namely:

$$d\mathbf{Y} = \partial \mathbf{Y} + \tilde{\mathbf{M}} d\mathbf{Y} \tag{A.28}$$

for some exogenous $\partial \mathbf{Y}$ satisfying $\mathbf{q}'\partial \mathbf{Y} = 0$, and some $\tilde{\mathbf{M}}$ satisfying $\mathbf{q}'\tilde{\mathbf{M}} = \mathbf{q}'$. In each case, constructing $\tilde{\mathbf{M}}$ requires knowledge of structural parameters beyond just iMPCs out of income.

## B.1 Monetary policy rule

Suppose that we are interested in the effects of fiscal policy shocks $d\mathbf{G}, d\mathbf{T}$, but that monetary policy follows a Taylor rule rather than a real interest rate rule.

**Real interest rate rule.** We first assume that the real interest rate $r_t$ responds to output directly, via the rule

$$dr_t = \phi_y dY_t \tag{A.29}$$

This could capture the response of monetary policy to the inflationary effects of government spending, as we will discuss further momentarily.

[A-15] Strictly speaking, since the initial calibration of the borrowing constraint $\underline{a} = 0$, this is a shock that forces agents to save a minimal amount for a temporary amount of time. Shocking a positive borrowing constraint would deliver the same effects, but would require recalibrating the steady state of the model.

We maintain our assumptions of equal incidence of income and taxes, so the consumption function in (A.25) specializes to $C_t = \mathcal{C}_t(\{Z_s, r_s\})$, whose derivative in the sequence space is $d\mathbf{C} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathbf{Z}$. Using the definition of post-tax income $d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}$, the market clearing condition $d\mathbf{Y} = d\mathbf{C} + d\mathbf{G}$, and the rule $d\mathbf{r} = \phi_Y d\mathbf{Y}$ from equation (A.29), we obtain:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T} + \left(\phi_y \mathbf{M}^r + \mathbf{M}\right) d\mathbf{Y}$$

Recall from section A.8 that any government plan $d\mathbf{G}, d\mathbf{T}$ must satisfy $\mathbf{q}' d\mathbf{T} = \mathbf{q}' d\mathbf{G} + B\mathbf{q}' d\mathbf{r}$, where here $d\mathbf{r}$ is endogenous to output. Because the change in real interest rates affects the government budget, the government cannot have a fully exogenous tax and spending plan: it must specify how it will endogenously raise taxes when interest rates increase. We therefore break the tax plan into two components, $d\mathbf{T} = d\mathbf{T}^{exo} + d\mathbf{T}^{endo}$, where the endogenous component of taxes satisfies $d\mathbf{T}^{endo} = \mathbf{J}^{T,r} d\mathbf{r}$, with $\mathbf{J}^{T,r}$ a Jacobian specifying how these taxes are levied, and satisfying $\mathbf{q}'\mathbf{J}^{T,r} = B\mathbf{q}'$, so that the response of endogenous taxes is always sufficient to cover the present value of additional interest on the public debt due to the change in $r_t$.

In this new setting, the output response $d\mathbf{Y}$ to an exogenous change in fiscal policy $d\mathbf{T}^{exo}, d\mathbf{G}$ satisfying $\mathbf{q}' d\mathbf{T}^{exo} = \mathbf{q}' d\mathbf{G}$ is therefore given by:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T}^{exo} + \tilde{\mathbf{M}} d\mathbf{Y} \tag{A.30}$$

where $\tilde{\mathbf{M}} \equiv \phi_y \left(\mathbf{M}^r - \mathbf{M}\mathbf{J}^{T,r}\right) + \mathbf{M}$. Note that we still have

$$\mathbf{q}'\tilde{\mathbf{M}} = \phi_y \left(\mathbf{q}'\mathbf{M}^r - \mathbf{q}'\mathbf{M}\mathbf{J}^{T,r}\right) + \mathbf{q}'\mathbf{M} = \phi_y \left(A\mathbf{q}' - \mathbf{q}'\mathbf{J}^{T,r}\right) + \mathbf{q}' = \phi_y (A - B)\, \mathbf{q}' + \mathbf{q}' = \mathbf{q}'$$

To conclude, equation (A.30) takes the form in (A.28), with the partial equilibrium spending impulse $\partial\mathbf{Y} \equiv d\mathbf{G} - \mathbf{M} d\mathbf{T}^{exo}$ and the modified $\mathbf{M}$ matrix $\tilde{\mathbf{M}}$ reflecting, in addition to the spending response to income, the spending response to the higher interest rates generated by the endogenous response of monetary policy.

**Nominal interest rate rule with a simple Phillips curve.** We next assume that monetary policy follows a nominal interest rate rule that responds to inflation, as in a standard Taylor rule specification,

$$di_t = \phi_\pi d\pi_t \tag{A.31}$$

Now, the response of real interest rates $dr_t$ is given by $dr_t = di_t - d\pi_{t+1}$, or in vector form

$$d\mathbf{r} = (\phi_\pi \mathbf{I} - \mathbf{F})\, d\boldsymbol{\pi} \tag{A.32}$$

where $\mathbf{F}$ is the lead operator. We also assume that the Phillips curve is given by (A.12), except for the fact that $C_t^*$ is replaced by aggregate consumption $C_t$.[A-16] After substituting the equilibrium

[A-16] Intuitively, in this case, the union treats all its members as if they had the average level of consumption $C_t$ from the perspective of evaluating the wealth effect on labor supply. We assume this here for convenience in order to avoid an relations $Y_t = N_t$, $C_t = Y_t - G_t$ and $\frac{Z_t}{N_t} = 1 - \frac{T_t}{Y_t}$, we therefore have:

$$d\pi_t^w = \kappa^w \left( \frac{1}{\phi Y} dY_t + \frac{1}{\sigma C} (dY_t - dG_t) + \frac{1}{Y} \frac{1}{1 - T/Y} \left( dT_t - \frac{T}{Y} dY_t \right) \right) + \beta d\pi_{t+1}^w$$

This can further be written as:

$$\begin{aligned} d\pi_t^w &= \kappa^w \frac{1}{\phi Y} \left( \left( 1 + \frac{\phi Y}{\sigma C} - \phi \frac{T/Y}{1 - T/Y} \right) dY_t - \frac{\phi Y}{\sigma C} dG_t + \phi \frac{dT_t}{1 - T/Y} \right) + \beta d\pi_{t+1}^w \\ &= \widetilde{\kappa}^w \left( dY_t - dY_t^n \right) + \beta d\pi_{t+1}^w \end{aligned} \tag{A.33}$$

where $\widetilde{\kappa}^w \equiv \kappa^w \frac{1 + \frac{\phi Y}{\sigma C} - \phi \frac{\frac{T}{Y}}{1 - T/Y}}{\phi Y}$ is the slope of the wage Phillips curve in terms of output, and $dY_t^n$, the natural level of output, is defined as:

$$dY_t^n \equiv \frac{\frac{\phi Y}{\sigma C}}{1 + \frac{\phi Y}{\sigma C} - \phi \frac{\frac{T}{Y}}{1 - T/Y}} dG_t - \frac{\frac{\phi}{1 - T/Y}}{1 + \frac{\phi Y}{\sigma C} - \phi \frac{\frac{T}{Y}}{1 - T/Y}} dT_t \tag{A.34}$$

In this case, the natural level of output only depends on contemporaneous government spending and taxes. This reflects the standard neoclassical forces (Aiyagari et al. 1992, Baxter and King 1993): there is a multiplier on government spending due to the standard wealth effect on labor supply, and a negative force from higher taxes due to the distortionary effects of taxation. Given (A.34), the natural level of output $d\mathbf{Y}^n$ is exogenously determined by $\{d\mathbf{G}, d\mathbf{T}\}$. Since productivity is constant, price and wage inflation are equal, $\pi_t = \pi_t^w$, so that (A.33) reads, in vector form, $d\boldsymbol{\pi} = \widetilde{\kappa}^w \left( d\mathbf{Y} - d\mathbf{Y}^n \right) + \beta \mathbf{F} d\boldsymbol{\pi}$. Since the operator norm $\|\beta \mathbf{F}\|$ is equal to $\beta < 1$, $\mathbf{I} - \beta \mathbf{F}$ is invertible with inverse $\sum_{k \geq 0} \beta^k \mathbf{F}^k$, and (A.33) reads simply

$$d\boldsymbol{\pi} = \mathbf{K}^w \left( d\mathbf{Y} - d\mathbf{Y}^n \right) \tag{A.35}$$

where

$$\mathbf{K}^w \equiv \widetilde{\kappa}^w \left( \sum_{k \geq 0} \beta^k \mathbf{F}^k \right) = \widetilde{\kappa}^w \begin{pmatrix} 1 & \beta & \beta^2 & \ddots \\ 0 & 1 & \beta & \ddots \\ 0 & 0 & 1 & \ddots \\ & \ddots & \ddots & \ddots \end{pmatrix}$$

is the standard expression for the Calvo–Rotemberg Phillips curve Jacobian (see e.g. Auclert, Rigato, Rognlie and Straub 2024b).

Combining (A.32) and (A.35) gives the endogenous response of the real interest rate under a nominal interest rate rule,

$$d\mathbf{r} = \left( \phi_\pi \mathbf{I} - \mathbf{F} \right) \mathbf{K}^w \left( d\mathbf{Y} - d\mathbf{Y}^n \right)$$

This is similar to (A.29), but takes into account the direct effect of government spending and dis-

additional fixed point with an endogenous real interest rate that would otherwise be required to solve for $C_t^*$.

tortionary taxation on inflation via the natural level of output. The rest of the analysis is therefore similar, leading to a fixed point in output as in (A.28), with $\partial \mathbf{Y}$ including an additional effect from the changing natural level of output $d\mathbf{Y}^n$.

### B.2 Nominal bonds

Suppose that, instead of trading real bonds, we modify the model so that households trade nominal one-period bonds instead, as in Auclert (2019), Angeletos et al. (2023) and Kaplan et al. (2023). Monetary policy continues to implement the constant-real interest rate rule $r_t = r$ for $t \geq 0$ onward, but now at time 0 the real return on existing bond positions is $\frac{1+r_{ss}}{1+\pi_0}$, reflecting the erosion of returns from inflation relative to its expected steady state value of 0. This implies two main changes to the equations of the model.

First, fiscal policy shocks $\{d\mathbf{T}, d\mathbf{G}\}$ now satisfy $\mathbf{q}'d\mathbf{T} + Bd\pi_0 = \mathbf{q}'d\mathbf{G}$: unexpected inflation acts as a tax and relaxes the government budget constraint. As in section B.1, the fiscal rule needs to specify how the government's tax and spending plan will respond to any change in inflation. Again, we do this by assuming that the response would fall on transfers, and split $d\mathbf{T} = d\mathbf{T}^{exo} + d\mathbf{T}^{endo}$, where $d\mathbf{T}^{endo} = -\mathbf{j}^{T,\pi}d\pi_0$ with $\mathbf{q}'\mathbf{j}^{T,\pi} = B$, so that the response of endogenous transfers is sufficient to exhaust the present-value gain from unexpected inflation.

Second, households experience a capital loss from unexpected inflation that affects their consumption according to the vector of MPCs out of capital gains $\mathbf{m}^{cap}$ (see section 6). In particular, consumption is now given by

$$d\mathbf{C} = \mathbf{M}d\mathbf{Z} - \mathbf{m}^{cap}Bd\pi_0$$

The rest of the environemnt is as in section 2.2. This implies in particular $d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}$ and $d\mathbf{Y} = d\mathbf{C} + d\mathbf{G}$.

In this environment, the impact response of inflation is needed to characterize the effect of the fiscal shocks $\{d\mathbf{T}, d\mathbf{G}\}$. Given (A.35), and setting $\phi = 0$ for simplicity so that we assume away the neoclassical forces discussed in section B.1, we obtain

$$d\pi_0 = \mathbf{e}_0'\mathbf{K}^w\left(d\mathbf{Y} - d\mathbf{Y}^n\right) = \mathbf{k}^{w\prime}d\mathbf{Y} \tag{A.36}$$

where $\mathbf{k}^{w\prime} \equiv \mathbf{e}_0'\mathbf{K}^w = \widetilde{\kappa}^w \begin{pmatrix} 1 & \beta & \beta^2 & \cdots \end{pmatrix}$ is the impact response of inflation of an increase in the path of output $d\mathbf{Y}$ via the Phillips curve. Combining these equations, we obtain the following modification of the intertemporal Keynesian Cross under nominal bonds. First, we have:

$$\begin{aligned} d\mathbf{Y} &= \mathbf{M}\left(d\mathbf{Y} - d\mathbf{T}^{exo} + \mathbf{j}^{T,\pi}d\pi_0\right) - B\mathbf{m}^{cap}d\pi_0 + d\mathbf{G} \\ &= \mathbf{M}\left(d\mathbf{Y} - d\mathbf{T}^{exo}\right) + \left(\mathbf{M}\mathbf{j}^{T,\pi} - B\mathbf{m}^{cap}\right)d\pi_0 + d\mathbf{G} \end{aligned} \tag{A.37}$$

The first and third term are standard. The second term in (A.37) reflects what happens to output when inflation rises: on the one hand, it leads to fiscal transfers with time path $\mathbf{j}^{T,\pi}$, which raise demand according to $\mathbf{M}$; on the other it directly lowers households' real wealth and lowers demand according to $\mathbf{m}^{cap}$. Overall, we have $\mathbf{q}'\left(\mathbf{M}\mathbf{j}^{T,\pi} - B\mathbf{m}^{cap}\right) = 0$, so the present value of spending is unaffected by inflation, but, provided the fiscal response to inflation embedded in $\mathbf{j}^{T,\pi}$ is sufficiently frontloaded, this term will generally be positive early on and negative later on. Combining (A.37) with (A.36) we obtain

$$d\mathbf{Y} = \underbrace{\left(\mathbf{M} + \left(\mathbf{M}\mathbf{j}^{T,\pi} - B\mathbf{m}^{cap}\right)\mathbf{k}^{w\prime}\right)}_{\equiv\tilde{\mathbf{M}}} d\mathbf{Y} + \underbrace{d\mathbf{G} - \mathbf{M}d\mathbf{T}^{exo}}_{\equiv\partial\mathbf{Y}}$$

which has the form in (A.28), with $\mathbf{q}'\tilde{\mathbf{M}} = \mathbf{q}'$. On balance, nominal bonds therefore just alter the $\mathbf{M}$ matrix by reflecting the influence of unexpected inflation on demand via the direct impact of eroding nominal wealth and the indirect impact through the fiscal response. Constructing $\tilde{\mathbf{M}}$ requires knowledge of the MPC out of capital gains $\mathbf{m}^{cap}$, as well as the endogenous response of transfers to inflation $\mathbf{j}^{T,\pi}$. This is a larger informational requirement than in the case of real bonds.

### B.3 Sticky prices

To explore the role of sticky prices, we take the limit of the model where unions reset wages perfectly flexibly, that is $\kappa^w = \infty$ in (A.11), so that we have:

$$\int N_t \left\{ v'\left(n_{it}\right) - \frac{\varepsilon - 1}{\varepsilon}\frac{\partial z_{it}}{\partial n_{it}} u'\left(c_{it}\right)\right\} di = 0 \tag{A.38}$$

Instead, assume that the nominal rigidity is in prices, with inflation following a standard Calvo Phillips curve,

$$\pi_t = \kappa^p w_t + \frac{1}{1+r}\pi_{t+1}$$

Faced with demand $Y_t$, firms hire labor $N_t = Y_t$ and earn profits

$$\Pi_t = Y_t - w_t N_t$$

Suppose that profits are distributed according to a rule: an agent with idiosyncratic ability $e$ receives share $\overline{\Pi}(e)$ of profits, so that agent $i$'s date-$t$ pretax income is now given by[A-17]

$$y_{it} = e_{it} w_t N_t + \overline{\Pi}(e_{it})\Pi_t$$

Rewriting this equation and using our assumptions so far, we find that

$$y_{it} = Y_t\left(w_t e_{it} + (1 - w_t)\,\overline{\Pi}(e_{it})\right)$$

In the sticky-wage, flexible-price model, the real wage is always equal to 1 and the share of aggregate income $Y_t$ going to agent type $i$ depends linearly on $e_{it}$, i.e. $y_{it} = Y_t e_{it}$. Here, with flexible

[A-17] The other common way to attribute profits to households is by allowing households to trade firms' shares. We make this assumption in sections (6) and (7).

wage but sticky price instead, the real wage fluctuates, and this affects the way in which income is distributed. In booms, when the real wage is large, income is split more according to ability $e_{it}$, while in busts the distribution of profits $\overline{\Pi}(e_{it})$ matters more.

A useful benchmark case is the one where the distribution of profits is also linear in ability,

$$\overline{\Pi}(e) = e \tag{A.39}$$

In this case, pretax income is exactly the same as in the sticky wage model,

$$y_{it} = Y_t e_{it}$$

Thus, for the constant-$r$ case, one may reinterpret our sticky wage model as a model with sticky prices together with the distribution rule for profits in (A.39). In particular, individual post-tax income $z_{it} \equiv \tau_t (y_{it})^{1-\theta}$ is still given by equation (8), and so solely dependent on $\{Y_t - T_t\}$, and the IKC from the main text (13) still characterizes the equilibrium response $\{dY_t\}$ to a change in fiscal policy $\{dG_t, dT_t\}$. Given a solution for these real quantities, we can then solve for the path for the real wage $\{w_t\}$ that guarantees that (A.38) holds at every $t$. This is analogous to how we solve for the path for the nominal wage $\{W_t\}$ after solving for real quantities in the sticky-wage, flexible-price model of section 2.

### B.4 Endogenous labor supply with GHH preferences

To explore the role of endogenous labor supply, we maintain our assumption of sticky prices from section B.3, but we now assume that agents choose hours flexibly and that they have GHH preferences as in Greenwood, Hercowitz and Huffman (1988).[A-18] We focus, for simplicity of the argument, of the case of the one-account model; other models of the general class introduced in section A.1 can be treated similarly. In addition to a rule $\overline{\Pi}(e_{it})$ for the distribution of aggregate profits $\Pi_t$ by ability, we assume a rule $\overline{T}(e_{it})$ for the distribution of aggregate taxes $T_t$ by ability. The Bellman equation for this model is then:

$$\begin{aligned} V_t(e_{it}, a_{it-1}) \quad &= \quad \max_{c_{it}, n_{it}} u(c_{it} - v(n_{it})) + \beta \mathbb{E}\left[V_{t+1}(e_{it+1}, a_{it}) | e_{it}\right] \\ \text{s.t.} \quad & c_{it} + a_{it} = (1 + r_{t-1}) a_{it-1} + e_{it} w_t n_{it} - T_t \overline{T}(e_{it}) + \Pi_t \overline{\Pi}(e_{it}) \\ & a_{it} \geq 0 \end{aligned} \tag{A.40}$$

The first order condition for hours is:

$$v'(n_{it}) = e_{it} w_t$$

[A-18] Away from GHH preferences, with general preferences $U(c, n)$ over consumption and labor, we can no longer reduce to a single fixed point in output. Instead, the model generates an aggregate labor supply function $\mathcal{N}_t$ in addition to an aggregate consumption function $\mathcal{C}_t$. See Auclert et al. (2023a) for a treatment of this case.

which leads to an hours choice $n_{it} = (v')^{-1}(e_{it}w_t)$. Substituting this in the the Bellman equation, we obtain:

$$\begin{aligned} V_t(e_{it}, a_{it-1}) &= \max_{c_{it}} u\left(c_{it} - v\left((v')^{-1}(e_{it}w_t)\right)\right) + \beta\mathbb{E}\left[V_{t+1}(e_{it+1}, a_{it})|e_{it}\right] \\ \text{s.t.} &\quad c_{it} + a_{it} = (1+r_{t-1})a_{it-1} + e_{it}w_t(v')^{-1}(e_{it}w_t) - T_t\overline{T}(e_{it}) + \Pi_t\overline{\Pi}(e_{it}) \\ &\quad a_{it} \geq 0 \end{aligned}$$

This has the form described in section A.1, with an ability- and time- varying utility function, and aggregate inputs $\mathbf{X}_t \equiv \{w_t, \Pi_t, T_t\}$. Applying the arguments of that section, we find that the model generates an aggregate consumption function $\mathcal{C}_t(\{w_t, \Pi_t, T_t\})$.

As in section B.3, profits are given by $\Pi_t = Y_t - w_tN_t = (1-w_t)Y_t$, and price inflation by $\pi_t = \kappa^p w_t + \frac{1}{1+r}\pi_{t+1}$. Moreover, the labor market clearing condition is:

$$\sum_{e_{it}} \pi(e_{it})(v')^{-1}(e_{it}w_t) = Y_t$$

which gives $w_t(Y_t)$ as a static function of $Y_t$. Substituting in these relations, we can define an aggregate consumption $\tilde{\mathcal{C}}_t(Y_t, T_t) \equiv \mathcal{C}_t(\{w_t(Y_t), (1-w_t(Y_t))Y_t, T_t\})$. The goods market clearing condition reads

$$\tilde{\mathcal{C}}_t(\{Y_t, T_t\}) + G_t = Y_t$$

Differentiating and defining $\mathbf{M}$ and $\mathbf{M}^T$ via $M_{ts} \equiv \frac{\partial\tilde{\mathcal{C}}_t}{\partial Y_s}$ and $M^T_{ts} \equiv \frac{\partial\tilde{\mathcal{C}}_t}{\partial T_s}$, we obtain the generalized intertemporal Keynesian cross

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^T d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

which takes the form in (A.28) with $\partial\mathbf{Y} \equiv d\mathbf{G} - \mathbf{M}^T d\mathbf{T}$.

### B.5 Durable goods

Here, we amend our framework to include durable spending. We then show that the model generates an intertemporal Keynesian cross provided the consumption function includes both nondurable and durable expenditure, as claimed in section 3.1.

**Model with durable goods.** We introduce durables in the simplest possible way, by assuming homothetic durable demand and perfect collateralizability. Considering the one-account model for ease of notation, and anticipating a constant-$r$ monetary policy rule, the household problem is now:

$$
\begin{aligned}
\max \quad & \mathbb{E}\left[\sum_{t\geq 0}\beta^t\left\{u\left(c_{it}\right)+\kappa u\left(d_{it}\right)\right\}\right] \\
& c_{it}+d_{it}-\left(1-\delta_D\right)d_{it-1}+a_{it}=z_{it}+\left(1+r\right)a_{it-1} \\
& a_{it}+\frac{1-\delta_D}{1+r}d_{it}\geq 0
\end{aligned}
$$

where $z_{it}$ is still taken as given and determined by labor demand in general equilibrium.

Observe that households can borrow against the non-depreciated component of the next period durable stock. Redefining the overall asset position as:

$$w_{it}\equiv a_{it}+\frac{1-\delta_D}{1+r}d_{it}$$

the problem rewrites as

$$
\begin{aligned}
\max \quad & \mathbb{E}\left[\sum_{t\geq 0}\beta^t\left\{u\left(c_{it}\right)+\kappa u\left(d_{it}\right)\right\}\right] \\
& c_{it}+\frac{r+\delta_D}{1+r}d_{it}+w_{it}=z_{it}+\left(1+r\right)w_{it-1} \\
& w_{it}\geq 0
\end{aligned}
$$

where the user cost of durables $\frac{r+\delta_D}{1+r}$ appears. In this formulation, no matter whether the constraint on $w_{it}$ is binding or not, there is a unique first order condition for the stock of durables $d_{it}$ relative to consumption $c_{it}$ that applies to every consumer, namely

$$\kappa u'\left(d_{it}\right)=u'\left(c_{it}\right)\left(\frac{r+\delta_D}{1+r}\right) \tag{A.41}$$

Equation (A.41) implies that the durable stock is a constant fraction of nondurable consumption at all times and for every consumer: $d_{it}=vc_{it}$ where $v=\left(u'\right)^{-1}\left(\frac{r+\delta_D}{1+r}\frac{1}{\kappa}\right)$. Further, given an initial level of wealth $w_{-1}$ and a stochastic process for $z_{it}$, if we let $c_{it}^{ND}$ be the path for nondurable consumption generated by our main model without durables, then the path for nondurable consumption in the model with durables is given, in every state and date, by $c_{it}=\frac{c_{it}^{ND}}{1+\frac{r+\delta}{1+r}v}$. Total expenditures $x_{it}\equiv c_{it}+d_{it}-\left(1-\delta_D\right)d_{it-1}$ in the enlarged model are therefore a simple lagged transformation of nondurable expenditures in the baseline model:

$$x_{it}=\frac{1+v}{1+\frac{r+\delta_D}{1+r}v}c_{it}^{ND}-\frac{\left(1-\delta_D\right)v}{1+\frac{r+\delta_D}{1+r}v}c_{it-1}^{ND}$$

Following the argument in section A.1, in the aggregate this behavior defines an expenditure function $\mathcal{X}_t\left(\left\{Z_s\right\}\right)$. Further, we have that the impact marginal propensity for expenditure (MPX)

$M_{00} = \frac{\partial \mathcal{X}_0}{\partial Z_0}$, is given by $\left(1 - s + s\frac{1+r}{r+\delta_D}\right)\frac{\partial \mathcal{C}_0^{ND}}{\partial Z_0}$, where $1 - s \equiv \frac{1}{1+\frac{r+\delta_D}{1+r}v}$ is the share $c_{it}/c_{it}^{ND}$ of nondurable spending in total notional consumption. This is the Laibson, Maxted and Moll (2022) formula to convert the impact "notional MPCs" from a model with only nondurables, $\frac{\partial \mathcal{C}_0^{ND}}{\partial Z_0}$, to the impact MPX.

**Intertemporal Keynesian cross with durable goods.** On the production side, we maintain our assumption that firms produce a unique good out of labor. The resource constraint for the economy is now:

$$G_t + \mathcal{X}_t\left(\{Y_s - T_s\}\right) = Y_t \tag{A.42}$$

Totally differentiating (A.42), we obtain the intertemporal Keynesian cross from the main text (13), which has the form (A.28) with $\partial \mathbf{Y} \equiv d\mathbf{G} - \mathbf{M}d\mathbf{Y}$, and where $\mathbf{M}$ now has elements $M_{t,s} \equiv \frac{\partial \mathcal{X}_t}{\partial Z_s}$, so that it includes total expenditures on both nondurables and durable goods, as claimed in section 3.1.[A-19]

## B.6 Investment

We now consider how the presence of investment with constant a constant real interest rate $r$ alters the intertemporal Keynesian cross. We first derive the existence of an aggregate investment function $\mathcal{I}_t\left(\{Y_s\}\right)$ and then discuss how this setting modifies the aggregate consumption function $\mathcal{C}_t\left(\{Y_s; T_s\}\right)$.

**Investment function.** We introduce a standard supply side with investment in appendix G.1 below. Here, we focus on the setting in section 2, in which prices are flexible, $\kappa^p = \infty$ and there are no markups $\mu = 1$. In that case, the economy's capital stock is determined as solution to the following fixed point. Given a path for real wages $\{w_t\}$, the economy's capital stock $K_t$ and labor supply $N_t$ solve

$$J_t(K_{t-1}) = \max_{K_t, N_t}\left\{F(K_{t-1}, N_t) - w_t N_t - \zeta\left(\frac{K_t}{K_{t-1}}\right)K_{t-1} + \frac{1}{1+r}J_{t+1}(K_t)\right\} \tag{A.43}$$

In turn, given a path for output $\{Y_t\}$, the equilibrium path for real wages $\{w_t\}$ must ensure that

$$F(K_{t-1}, N_t) = K_{t-1}^{\alpha} N_t^{1-\alpha} = Y_t \tag{A.44}$$

For given $K_{t-1}$, denote by $\mathcal{N}(K_{t-1}, Y_t) = Y_t^{1/(1-\alpha)} K_{t-1}^{-\alpha/(1-\alpha)}$ the level of labor compatible with (A.44). We assume that there exists a unique equilibrium path for the capital stock $\{K_{t-1}\}$. We characterize this path as follows:

[A-19] In an earlier version of this paper (Auclert, Rognlie and Straub 2018), we calibrated this enlarged model to match the Norwegian evidence on $M_{t0}$, and used this model to extrapolate to other columns of the $\mathbf{M}$ matrix. This extrapolation gives an outcome very similar to the one from our one-account model. The main difference is that spending is not as elevated in the year immediately after the income receipt, as households decumulate some of their durables.

**Lemma 5.** *$\{K_t\}$ is the equilibrium path of capital if and only if $\{K_t\}$ solves the following problem, in which the real wage does not appear:*

$$\mathcal{J}_t(K_{t-1}) = \max_{K_t} \left\{ \mathcal{F}(K_{t-1}, Y_t) - \zeta\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} + \frac{1}{1+r}\mathcal{J}_{t+1}(K_t) \right\} \tag{A.45}$$

*where we define*

$$\mathcal{F}(K, Y) \equiv \alpha Y \log K$$

*Proof.* The only difference between (A.43) and (A.45) are the terms $F(K_{t-1}, N_t) - w_t N_t$ and $\mathcal{F}(K_{t-1}, Y_t)$. The derivative of the former with respect to capital, evaluated on the equilibrium path, is

$$F_K(K_{t-1}, \mathcal{N}(K_{t-1}, Y_t)) = \alpha Y_t / K_{t-1}$$

which is equal to $\mathcal{F}_K(K_{t-1}, Y_t)$. Thus, any equilibrium path satisfies the first order conditions of (A.45), and any solution to (A.45) satisfies the first order conditions of (A.43). Since there is a unique equilibrium path, this also implies that there can only be a single solution to (A.45). □

Lemma 5 is helpful since it immediately implies that there is a capital function $\mathcal{K}_t(\{Y_s\})$ mapping paths $\{Y_s\}$ to the equilibrium path of capital $\{K_t\}$, and therefore also an investment function $\mathcal{I}_t(\{Y_s\}) \equiv \mathcal{K}_t(\{Y_s\}) - (1-\delta)\mathcal{K}_{t-1}(\{Y_s\})$.

**Modified consumption function.** Given the presence of investment, shocks induce valuation effects as discussed in section 6.1. At date 0, the new value of shares in firms, including dividends, is $p_0 + d_0 = J_0(K_{-1})$. Substituting the capital function $\mathcal{K}_t(\{Y_s\})$ and the labor function $\mathcal{N}_t(\{Y_s\})$ into (A.43), we see that $J_0(K_{-1})$ is a function $\mathcal{J}(\{Y_s\})$. Observing that aggregate labor income $w_s N_s$ is simply $(1-\alpha) Y_s$, and substituting these relations into the consumption function inclusive of valuation in (33), we now have

$$C_t = \mathcal{C}_t(\{(1-\alpha) Y_s - T_s\}, \mathcal{J}(\{Y_s\})) \equiv \tilde{\mathcal{C}}_t(\{Y_s; T_s\}).$$

where the dependence of $\tilde{\mathcal{C}}$ on $Y_s$ is both through the effect that $Y_s$ has on labor income $w_s N_s$, and through the effect it has on the initial value of capital $\mathcal{J}$.

**Intertemporal Keynesian cross with investment.** Having derived the investment function as well as the modified consumption function, the goods market clearing condition now reads

$$Y_t = G_t + \tilde{\mathcal{C}}_t(\{Y_s; T_s\}) + \mathcal{I}_t(\{Y_s\})$$

Differentiating this equation, we find an intertemporal Keynesian cross equation (A.28) where $\partial\mathbf{Y} \equiv d\mathbf{G} - \mathbf{M}^T d\mathbf{T}$ with the elements of $\mathbf{M}^T$ being given by $M^T_{t,s} = \frac{\partial \tilde{\mathcal{C}}_t}{\partial T_s}$ and the elements of $\tilde{\mathbf{M}}$ being given by $\tilde{M}_{t,s} = \frac{\partial \tilde{\mathcal{C}}_t}{\partial Y_s} + \frac{\partial \mathcal{I}_t}{\partial Y_s}$.

# C Appendix to section 3

## C.1 Proof of Lemma 1

Define

$$E_c^t\left(\mathbf{s},\boldsymbol{\omega}_-;\{Z_s\}\right)\equiv\mathbb{E}_0\left[c_t|(\mathbf{s}_0,\boldsymbol{\omega}_{-1})=(\mathbf{s},\boldsymbol{\omega}_-)\right]$$

as the expectation at date 0 of consumption at time $t$ for an agent that is in state $(\mathbf{s},\boldsymbol{\omega}_-)$ at date 0 and follows the policy induced by the sequence $\{Z_s\}$. Since, by perfect foresight, the average date-0 expectation of date-$t$ consumption will equal actual aggregate consumption at date $t$, we have:

$$C_t=\int E_c^t\left(\mathbf{s},\boldsymbol{\omega}_-\right)d\mu\left(\mathbf{s},\boldsymbol{\omega}_-\right)$$

where $\mu$ is the steady-state distribution. It follows that:

$$M_{t0}=\frac{\partial C_t}{\partial Z_0}=\int\frac{\partial E_c^t\left(\mathbf{s},\boldsymbol{\omega}_-\right)}{\partial Z_0}d\mu\left(\mathbf{s},\boldsymbol{\omega}_-\right)$$

Replacing with more familiar notation for $E_c^t$, and using the fact that $Z_0$ only enters agent $i$'s problem through its $z_{i0}$, with $\frac{\partial z_{i0}}{\partial Z_0}=\frac{z_{i0}}{Z_0}=\frac{z_{i0}}{\int z_{\iota 0}d\iota}$, we obtain:

$$M_{t0}=\int\frac{\partial\mathbb{E}_0\left[c_{it}\right]}{\partial Z_0}di=\int\frac{\partial\mathbb{E}_0\left[c_{it}\right]}{\partial z_{0i}}\frac{\partial z_{i0}}{\partial Z_0}di=\int\frac{z_{i0}}{\int z_{\iota 0}d\iota}\frac{\partial\mathbb{E}_0\left[c_{it}\right]}{\partial z_{0i}}di$$

This delivers Lemma 1.

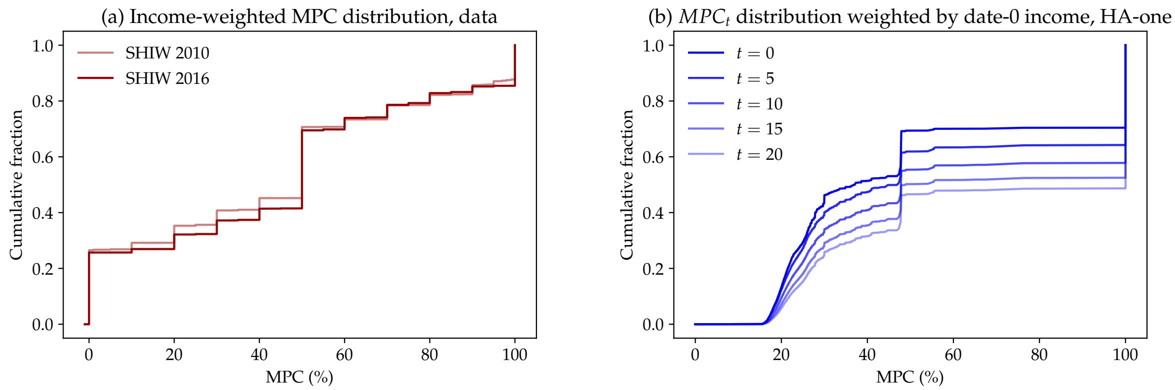
## C.2 Evidence from Norwegian administrative data

Our estimates on Norwegian iMPCs were generously provided to us by Andreas Fagereng, Martin Holm and Gisle Natvik. In Fagereng et al. (2021), they combine individual-level administrative income data and household-level wealth data from the Norwegian population, and residually impute a household-level consumption measure using a budget constraint approach. This is therefore a comprehensive measure of household expenditure, including durable and housing expenditures. However, the authors drop from their sample all households who record a housing market transaction, so that their iMPC estimates can be interpreted as including consumption and non-housing durable expenditures only.

The paper provides convincing evidence that the sample of gamblers is not selected: 70 percent of the population gambles, the population of winners is not significantly different from the rest of the population on observable characteristics including their consumption-income covariance over time, and gambling prizes are not predictable by prior household characteristics (Tables 1 and 2). To further limit the concern that iMPC estimates reflect the behavior of serial gamblers, the sample is limited to households who win only once.

The authors provided us with income-weighted estimates of regression (21). The regression

Figure C.1: Income-weighted MPC distributions: data and HA-one model




includes all lottery wins below \$150,000, and most prizes are below \$20,000. As we discuss in footnote 21, their MPC estimates for a sample restricted to small gains are much larger than the full sample estimates, imprecisely estimated, and do not sum to one, so we prefer to use these full-sample estimates.

## C.3 Evidence from the Italian Survey of Household Income and Wealth

The Italian Survey of Household Income and Wealth (SHIW) is a biannual survey, publicly available on the Bank of Italy website. In 2016, survey respondents were asked:

> "Imagine you unexpectedly receive a refund equal to the household's monthly income. How much of the sum would you save and how much would you spend? Indicate the percentage saved and the percentage spent."

In 2010, the same question was asked, except that the survey mentioned a "reimbursement" rather than a "refund". The distribution of answers to this question is similar to that of the the 2012 survey which specified a timeframe "over the next 12 months" with a slightly different wording. As a result, these answers are typically interpreted as annual MPCs. (The 2014 survey instead included a retrospective question about spending of the 2014 "Renzi bonus".)

We drop observations with zero or negative income, and are left with 7,936 observations in the 2010 survey and 7,367 observations in the 2016 survey.

**Distributions of MPCs.** Panel (a) of figure C.1 displays the cumulative density functions of the income-weighted distribution of MPCs in the 2010 and 2016 SHIW. As is apparent, these distributions are quite similar, despite being measured six years apart, which we take to support our choice of treating the 2016 distribution as corresponding to the steady state.[A-20]

[A-20] The similarity in these distributions over time also suggests that one possible source of aggregate state-dependence—different MPC distributions in different macroeconomic conditions—is not present.

**Construction of a lower bound.** From the 2016 survey, we have a distribution of self-reported MPCs $MPC_i$ as well as income net of taxes $z_i$. We can therefore construct $M_{00} = \mathbb{E}_I\left[\frac{z_i}{\mathbb{E}_I[z_i]} MPC_i\right]$ directly from the data.

Next, we make further assumptions on the primitives in appendix A.1. In particular, we assume that there is only a single account, i.e. $L = 1$, and that both income $z_{it}$ and lagged assets $a_{it-1}$ only appear in the date-$t$ optimization problem (A.1) in the budget constraint $c_{it} + a_{it} = (1+r)a_{it-1} + z_{it}$.[A-21] These assumptions hold for all models in section 4 except the two-account model.

Because $(1+r)a_{it-1}$ and $z_{it}$ enter the problem interchangeably, the derivatives of both the consumption $c_{it}$ and asset $a_{it}$ choices with respect to $(1+r)a_{it-1}$ and $z_{it}$ must be the same. Defining these (potentially time-varying and state-contingent) derivatives in the aggregate steady state to be $MPC_{ti}$ and $MPS_{ti} \equiv 1 - MPC_{ti}$, respectively, it follows that if an individual $i$ receives a unit income shock at date 0, then his assets at date 0 will increase by $MPS_{0i}$. Going into date 1, he will have $(1+r)MPS_{0i}$ more resources, leading to higher savings in assets of $(1+r)MPC_{0i} \cdot MPS_{1i}$, and so on. In general, his asset balance at date $t$, conditional on the same sequence of shocks, will increase by

$$da_{it} = (1+r)^t MPS_{0i} \cdot MPS_{1i} \cdots MPS_{ti}$$

Aggregating across all individuals $i$, who each receive income shocks $\frac{z_{i0}}{\int z_{\iota 0} d\iota}$ in response to an aggregate unit income shock, and taking expectations to aggregate over all realizations of shocks, we have

$$dA_t = (1+r)^t \int \frac{z_{i0}}{\int z_{\iota 0} d\iota} \mathbb{E}_0[MPS_{0i} \cdot MPS_{1i} \cdots MPS_{ti}] di \tag{A.46}$$

We would like to bound (A.46) from above, to obtain a lower bound on cumulative consumption.

To do so, we make the following intuitive assumption: the date-0 marginal propensity to save, $MPS_{0i}$, is more closely positively related to after tax income at date 0, $z_{i0}$, than the marginal propensity to save at a later date, $MPS_{ti}$. Formally, the date-0-income-weighted distribution of $MPS_{0i}$ first order stochastically dominates the income-weighted distribution of $MPS_{ti}$, or for any $t > 0$ and $m > 0$,

$$\mathbb{P}_I\left(\frac{z_{i0}}{\int z_{\iota 0} d\iota} MPS_{0i} > m\right) > \mathbb{P}_I\left(\frac{z_{i0}}{\int z_{\iota 0} d\iota} MPS_{ti} > m\right) \tag{A.47}$$

Panel (b) of figure C.1 shows that this assumption is clearly satisfied in our HA-one model. The following result shows that this assumption is sufficient to establish a simple upper bound on $dA_t$.

**Proposition 10.** *Assume that* (A.47) *holds. Define* $\overline{dA_t}$ *as*

$$\overline{dA_t} \equiv (1+r)^t \int \frac{z_i}{\int z_\iota d\iota} (MPS_i)^{t+1} di \tag{A.48}$$

[A-21] This latter condition rules out, for instance, that lagged assets $a_{it-1}$ are complements to consumption in the utility function, or that they appear separately in another constraint on choices.

*then, for $t = 0$, $dA_0 = \overline{dA_0}$, and for $t > 0$, $dA_t \le \overline{dA_t}$, where $dA_t$ is as in (A.46).*

Note that the key object $\int \frac{z_i}{\int z_\iota d\iota} (MPS_i)^{t+1} di$ in (A.48) is the income-weighted cross-sectional average of $(MPS_i)^{t+1}$ in the aggregate steady state. We formally prove proposition 10 below.

The intuition behind proposition 10 is that if the cross-sectional distribution of $MPS_{ti}$ across individuals $i$, weighted by income at date 0, is the same at all dates $t$, then savings will be the highest if $MPS_{ti}$ remains the same for everyone—i.e. if the same households with a high marginal propensity to save at date 0 also have a high marginal propensity to save at dates $1, 2, \ldots$. (Inversely, savings are low if $MPS_{it}$ switches between households: if savers yesterday are spenders today, and vice versa, then little or no savings will persist past today.)

Generally, it will not be true that the distribution of $MPS_{it}$ weighted by income at date 0 is the same over time, but it is reasonable to assume that the distribution shifts to the left as $t$ increases (and so the $MPC_{it}$ distribution shifts to the right, as depicted in figure C.1(b)). Therefore, the upper bound for savings computed by using the $z_{i0}$-weighted distribution of $MPS_{i0}$ remains an upper bound, as the proposition shows.

Proposition 10 implies the following key corollary, which is the source of our lower bound on intertemporal MPCs.

**Corollary 2** (Bound on cumulative iMPCs)**.** *Define $\underline{dC_0} \equiv 1 - \overline{dA_0}$ and $\underline{dC_t} \equiv (1+r)\overline{dA_{t-1}} - \overline{dA_t}$ for $t > 0$. Then $dC_0 = \underline{dC_0}$, $dC_1 \ge \underline{dC_1}$, and for $t > 1$:*

$$\sum_{t'=0}^{t} (1+r)^{-t'} dC_{t'} \ge \sum_{t'=0}^{t} (1+r)^{-t'} \underline{dC_{t'}} \tag{A.49}$$

*Proof of corollary 2.* $dC_0 = \underline{dC_0}$ follows immediately from $dC_0 = 1 - dA_0$ and $dA_0 = \overline{dA_0}$ in proposition 10. Similarly, $dC_1 \ge \underline{dC_1}$ follows from $dC_1 = (1+r)dA_0 - dA_1 = (1+r)\overline{dA_0} - dA_1 \ge (1+r)\overline{dA_0} - \overline{dA_1} = \underline{dC_1}$.

For $t > 1$, we can sum the $dC_0 = 1 - dA_0$ and the discounted $dC_t = (1+r)dA_{t-1} - dA_t$ to obtain $\sum_{t'=0}^{t}(1+r)^{-t'} dC_{t'} = 1 - (1+r)^{-t} dA_t$, and similarly $\sum_{t'=0}^{t}(1+r)^{-t'} \underline{dC_{t'}} = 1 - (1+r)^{-t}\overline{dA_t}$. Combining these with $dA_t \le \overline{dA_t}$ gives (A.49), as desired. □

Note that when $t > 1$, (A.49) allows for $dC_t$ to be less than $\underline{dC_t}$, but only when $dC_{t'}$ commensurately exceeded $\underline{dC_{t'}}$ for some earlier $t'$. (Our focus in the main text is on the $t = 1$ case, where this is not an issue.)

Since $\underline{dC_t}$ are lower bounds on the cumulative consumption response out of a unit date-0 shock to aggregate income, they are lower bounds on cumulative iMPCs $M_{t0}$, and in the main text we denote them by $\underline{M_{t0}}$. Figure 1 reports $\underline{M_{t0}}$, which we compute using our the value $r = 5\%$ from our calibration.

*Proof of proposition 10.* The proposition requires a generalized version, proved in lemma 6 below, of what is sometimes known as the "rearrangement inequality".[A-22]

[A-22] The basic form of the discrete rearrangement inequality states that for any real numbers $x_1 \le \cdots \le x_n$ and $y_1 \le$

Define the random variables in lemma 6 to be $X_j = MPS_{ij-1}$ and $Y = MPS_{i0}$, where the probability measure $\mu$ is defined over all individuals $i$ and their histories, weighting by date-0 income. Under this measure, using (A.47), we have that $X_i \preceq Y$, and therefore can apply lemma 6 to obtain $\mathbb{E}[MPS_{i0} \cdots MPS_{it}] \leq \mathbb{E}[MPS_{i0}^{t+1}]$ for all $t$. This implies that for a measure unweighted by income, we have $\mathbb{E}[\frac{z_{i0}}{\int z_{\iota 0} d\iota} MPS_{i0} \cdots MPS_{it}] \leq \mathbb{E}[\frac{z_{i0}}{\int z_{\iota 0} d\iota} MPS_{i0}^{t+1}]$, where $\mathbb{E}$ takes both expectations for each individual and the cross-sectional average across them. Given (A.46) and (A.48), this is in turn equivalent to the bound in proposition 10, where we drop the 0 time subscript in $\frac{z_{i0}}{\int z_{\iota 0} d\iota} MPS_{i0}^{t+1}$ since its average is the same in all dates in the aggregate steady state. □

**Lemma 6.** *If $X_1, \ldots, X_n, Y$ are nonnegative random variables on some arbitrary probability space $(\Omega, \mathcal{A}, \mu)$, and $X_i \preceq Y$ in the sense of first-order stochastic dominance, then*

$$\mathbb{E}[X_1 \cdots X_n] \leq \mathbb{E}[Y^n] \tag{A.50}$$

*Proof.* To prove this lemma, first consider the case where the random variables all take values only in some finite set $a_1, \ldots, a_m$, which we put in strictly increasing order. Then for any $X_i$ we can write

$$X_i = \sum_{j=1}^{m} a_j \mathbf{1}_{X_i = a_j} = \sum_{j=1}^{m} \alpha_j \mathbf{1}_{X_i \geq a_j}$$

where $\alpha_1 \equiv a_1, \alpha_2 \equiv a_2 - a_1, \ldots, \alpha_m \equiv a_m - a_{m-1}$ (and similarly for $Y$).[A-23] We can then write

$$\begin{aligned}
\mathbb{E}[X_1 \cdots X_n] &= \mathbb{E}\left[\prod_{i=1}^{n}\left(\sum_{j=1}^{m} \alpha_j \mathbf{1}_{X_i \geq a_j}\right)\right] \\
&= \sum_{j_1=1}^{m}\sum_{j_2=1}^{m}\cdots\sum_{j_n=1}^{m}(\alpha_{j_1}\cdots\alpha_{j_n})\mathbb{E}[\mathbf{1}_{X_1 \geq a_{j_1}}\cdots\mathbf{1}_{X_n \geq a_{j_n}}] \\
&= \sum_{j_1=1}^{m}\sum_{j_2=1}^{m}\cdots\sum_{j_n=1}^{m}(\alpha_{j_1}\cdots\alpha_{j_n})\mu\left(\{X_1 \geq a_{j_1}\}\cap\cdots\cap\{X_n \geq a_{j_n}\}\right) \\
&\leq \sum_{j_1=1}^{m}\sum_{j_2=1}^{m}\cdots\sum_{j_n=1}^{m}(\alpha_{j_1}\cdots\alpha_{j_n})\mu\left(\{Y \geq \max_i j_i\}\right) \\
&= \sum_{j_1=1}^{m}\sum_{j_2=1}^{m}\cdots\sum_{j_n=1}^{m}(\alpha_{j_1}\cdots\alpha_{j_n})\mathbb{E}[\mathbf{1}_{Y \geq a_{j_1}}\cdots\mathbf{1}_{Y \geq a_{j_n}}] \\
&= \mathbb{E}\left[\prod_{i=1}^{n}\left(\sum_{j=1}^{m} \alpha_j \mathbf{1}_{Y \geq a_j}\right)\right] = \mathbb{E}[Y^n]
\end{aligned}$$

Here, the key inequality step is $\mu\left(\{X_1 \geq a_{j_1}\}\cap\cdots\cap\{X_n \geq a_{j_n}\}\right) \leq \mu\left(\{Y \geq \max_i j_i\}\right)$. This follows because trivially $\mu\left(\{X_1 \geq a_{j_1}\}\cap\cdots\cap\{X_n \geq a_{j_n}\}\right) \leq \mu(\{X_i \geq a_{j_i}\})$ for any $i$; and then by

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$\cdots \leq y_n$, we have $x_{\sigma(1)}y_1 + \cdots + x_{\sigma(n)}y_n \leq x_1 y_1 + \cdots + x_n y_n$ for any permutation $\sigma$ of $1, \ldots, n$. A special case is where we have the same sequence: $x_{\sigma(1)}x_1 + \cdots x_{\sigma(n)}x_n \leq x_1^2 + \cdots + x_n^2$. Lemma 6 applies this idea to the case of many random variables defined on a probability space, all dominated in distribution by a common random variable.

[A-23] This is sometimes called the "layer-cake representation" of a function.

the first-order stochastic dominance assumption, $\mu(\{X_i \geq a_{j_i}\}) \leq \mu(\{Y \geq a_{j_i}\})$, also for any $i$.

This proves (A.50) for the case of random variables that take finitely many values. For the general case, where $X_i$ and $Y$ can take any nonnegative values, let us define

$$X_i^{(N)}(\omega) \equiv \begin{cases} \frac{k}{N} & \text{if } \frac{k}{N} \leq X_i(\omega) < \frac{k+1}{N} \text{ for } k \in \{0, 1, \ldots, N^2 - 1\} \\ N & \text{if } X_i(\omega) \geq N \end{cases}$$

It is clear that $X_i^{(N)} \leq X_i$ for all $N$, and that $X_i^{(N)} \to X_i$ almost everywhere as $N \to \infty$. Defining $Y^N$ similarly, we also have $Y^{(N)} \leq Y$ and $Y^{(N)} \to Y$.

Given our result for random variables with finitely many values, we have

$$\mathbb{E}[X_1^{(N)} \cdots X_n^{(N)}] \leq \mathbb{E}[(Y^{(N)})^n]$$

for any $N$, and taking the limit as $N \to \infty$ and applying the dominated convergence theorem, we obtain (A.50). □

### C.4 Literature on the consumption response to anticipated income shocks

The existing evidence points to the presence of some, albeit modest, anticipation effects. For example, in their survey, Fuster et al. (2021) find that a few households would cut spending immediately in response to the news of a $500 loss one quarter ahead. Agarwal and Qian (2014) find evidence of a spending response between the announcement of a cash payout in Singapore and its disbursement two months afterward. Di Maggio et al. (2017) find some evidence of one-quarter-ahead new car spending in expectation of a predictable reduction in mortgage payments. By contrast, Kueng (2018) finds limited evidence of anticipation effects from the Alaska Permanent Fund news. The bulk of this evidence is for quarterly rather than yearly spending, and is too imprecise for us to confidently use as a model input.

## D Appendix to section 4

### D.1 Marginal value function propagation via policy derivative

We first prove a result about the propagation of shocks in heterogeneous-agent models that will be useful for the proofs that follow.

Consider a model with a single continuous endogenous state denoted by $a$ (for assets), and an exogenous state $\mathbf{s}$ following a discrete Markov chain $\Pi_{\mathbf{ss}'}$. Assume that the Bellman equation can be represented as:

$$V_t(\mathbf{s}, a_-) = \max_a F_t(\mathbf{s}, a_-, a) + \beta \mathbb{E}\left[V_{t+1}(\mathbf{s}', a) | \mathbf{s}\right] \tag{A.51}$$

for a certain function $F_t$. Let $a_t(\mathbf{s}, a_-)$ denote the policy function. In the steady state, the flow objective is $F_t = F$, the value function is $V_t = V$, and the policy is $a(\mathbf{s}, a_-)$.

Consider a perturbation to the steady state that leads to a new set of policies $\{dV_s(\mathbf{s}, a_-)\}$ but does not affect the value of $F$ at time $t$. Then, by the envelope theorem, we that:

$$dV_t(\mathbf{s}, a_-) = \beta\mathbb{E}\left[dV_{t+1}(\mathbf{s}', a(\mathbf{s}, a_-))|\mathbf{s}\right]$$

where the steady state policy $a(\mathbf{s}, a_-)$ is used. Differentiating both sides with respect to $a_-$, denoting $V_t' \equiv \frac{\partial V_t}{\partial a_-}$ and $a' \equiv \frac{\partial a}{\partial a_-}$, and using the chain rule and the symmetry of mixed partials, we obtain:[A-24]

$$dV_t'(\mathbf{s}, a_-) = \beta a'(\mathbf{s}, a_-)\,\mathbb{E}\left[dV_{t+1}'(\mathbf{s}', a(\mathbf{s}, a_-))|\mathbf{s}\right] \qquad \forall(\mathbf{s}, a_-) \tag{A.52}$$

Equation (A.52) shows that future shocks to the marginal value function propagate to earlier periods through discounting $\beta$ and the derivative of the steady-state policy $a'(\mathbf{s}, a_-)$. This result has important implications for the structure of sequence-space Jacobians under rational expectations. Two special cases are particularly useful.

First, consider a model without idiosyncratic shocks. Let $\bar{a} = a(\bar{a})$ denote the steady-state value of $a$. Then (A.52) implies

$$dV_t'(\bar{a}) = \beta\lambda dV_{t+1}'(\bar{a}) \tag{A.53}$$

where the scalar $\lambda \equiv a'(\bar{a})$ is the derivative of the steady-state policy at the steady-state level of assets.

Consider next a model with idiosyncratic shocks, but for which the steady-state is such that $a(\mathbf{s}, \bar{a}) = \bar{a}$ for all $\mathbf{s}$. This is the case, for instance, in the zero-liquidity model considered in section D.4, in which all agents are at the borrowing constraint in equilibrium. Then (A.52) implies that

$$dV_t'(\mathbf{s}, \bar{a}) = \beta\lambda_{\mathbf{s}}\mathbb{E}\left[dV_{t+1}'(\mathbf{s}', \bar{a})|\mathbf{s}\right] \qquad \forall\mathbf{s} \tag{A.54}$$

where, for each $\mathbf{s}$, the scalar $\lambda_{\mathbf{s}} \equiv a'(\mathbf{s}, \bar{a})$ is the derivative of the steady-state policy evaluated at the steady-state level of assets in state $\mathbf{s}$.

## D.2 Analytical models: RA, BU, TA and TABU

The intertemporal first-order condition for the unconstrained agent solving problem (22) is given by:

$$u'(c_t^u) = \beta(1+r)\,u'(c_{t+1}^u) + \chi'(a_t^u) \tag{A.55}$$

In a steady state, we must have

$$u'(c^u)(1 - \beta(1+r)) = \chi'(a^u) \tag{A.56}$$

[A-24] Letting $\theta$ be the perturbation to the steady state, we have $dV_t(\mathbf{s}, a_-) \equiv \frac{\partial V_t(\mathbf{s}, a_-)}{\partial\theta}d\theta$, and here we use the fact that $\frac{\partial}{\partial a_-}\frac{\partial V_t(\mathbf{s}, a_-)}{\partial\theta}d\theta = \frac{\partial}{\partial\theta}\frac{\partial V_t(\mathbf{s}, a_-)}{\partial a_-}d\theta = \frac{\partial}{\partial\theta}\frac{\partial}{\partial a_-}(V_{t+1}(\mathbf{s}, a(\mathbf{s}, a_-)))\,d\theta = a'(\mathbf{s}, a_-)\frac{\partial}{\partial\theta}\frac{\partial V_{t+1}(\mathbf{s}, a(\mathbf{s}, a_-))}{\partial a_-}d\theta = a'(\mathbf{s}, a_-)\frac{\partial}{\partial a_-}\frac{\partial V_{t+1}(\mathbf{s}, a(\mathbf{s}, a_-))}{\partial\theta}d\theta$.

as well as

$$c^u = Z + ra^u \tag{A.57}$$

Away from steady state, starting at an asset position of $a^u_{-1}$, the intertemporal budget constraint corresponding to (22) reads

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t c^u_t = \sum_{t=0}^{\infty}\frac{1}{(1+r)^t}Z_t + (1+r)\,a^u_{-1} \tag{A.58}$$

**RA model.** In the RA model, $\chi' = 0$ implies $\beta(1+r) = 1$ from (A.55), and $c^u_t = c^u$ from (A.55). $\mu = 0$ then implies that $c_t = c^u$. Plugging into (A.58), we obtain the level of $c^u$ and therefore the consumption function,

$$\mathcal{C}^{RA}_t(\{Z_s\}) = \frac{r}{1+r}\sum_{s=0}^{\infty}\frac{1}{(1+r)^s}Z_s + ra_{-1} = (1-\beta)\sum_{s=0}^{\infty}\beta^s Z_s + ra_{-1} \tag{A.59}$$

This implies the matrix of iMPCs $\mathbf{M}^{RA}$ in the main text, (23). Each column is constant, since the agent is a permanent-income consumer and thus consumes the constant annuity value of any increase in after-tax income. The asset function of the RA model solves can be derived by iterating forward on the relation (A.6), which gives:

$$\begin{aligned}
\mathcal{A}^{RA}_t(\{Z_s\}) &= \sum_{u=t+1}^{\infty}\beta^{u-t}\left\{\mathcal{C}_u(\{Z_s\}) - Z_u\right\} = \frac{\beta}{1-\beta}\mathcal{C}(\{Z_s\}) - \sum_{u=t+1}^{\infty}\beta^{u-t}Z_u \\
&= \beta\left(\sum_{s=0}^{\infty}\beta^s Z_s - \sum_{s=t+1}^{\infty}\beta^{s-(t+1)}Z_s + \frac{\beta}{1-\beta}ra_{-1}\right)
\end{aligned}$$

This equation the asset Jacobian, $A^{RA}_{t,s} = \beta\cdot\beta^s$ for $t \geq s$ and $A^{RA}_{t,s} = \beta^{s+1} - \beta^{s-t} = -\beta\cdot\beta^{s-(t+1)}\left(1-\beta^{t+1}\right)$ for $s \geq t+1$. In matrix form, this reads

$$\mathbf{A}^{RA} = \beta\begin{pmatrix}
1 & -(1-\beta) & -(1-\beta)\,\beta & -(1-\beta)\,\beta^2 & \cdots \\
1 & \beta & -\left(1-\beta^2\right) & -\left(1-\beta^2\right)\beta & \cdots \\
1 & \beta & \beta^2 & -\left(1-\beta^3\right) & \ddots \\
\vdots & \vdots & \vdots & \beta^3 & \ddots \\
 & & & &
\end{pmatrix} \tag{A.60}$$

This verifies $(\mathbf{I} - (1+r)\,\mathbf{L})\,\mathbf{A}^{RA} = \mathbf{I} - \mathbf{M}^{RA}$, or alternatively, $\mathbf{A}^{RA} = \mathbf{K}\left(\mathbf{I} - \mathbf{M}^{RA}\right)$, with $\mathbf{K}$ defined as in proposition 1.

**TA model.** In the TA model, we still have $\beta(1+r) = 1$ from (A.55), and $c_t^u = c^u$ from (A.55). Now, $c_t = (1-\mu)c^u + \mu c_t^c$ with $c_t^c = Y_t - T_t$. This implies the consumption function:

$$\mathcal{C}_t^{TA}(\{Z_s\}) = (1-\mu)\left\{\frac{r}{1+r}\sum_{s=0}^{\infty}\frac{1}{(1+r)^s}Z_s + ra_{-1}\right\} + \mu Z_t \tag{A.61}$$

The iMPC matrix $\mathbf{M}$ is therefore a convex combination of $\mathbf{M}^{RA}$, and the identity matrix $\mathbf{I}$, which captures the consumption response of the hand-to-mouth agents. This gives the expression for $\mathbf{M}^{TA}$ in the main text, (24).

For the asset Jacobian, we note that $a_t = (1-\mu)a_t^u$, which implies the asset function

$$\mathcal{A}_t^{TA}(\{Z_s\}) = \beta(1-\mu)\left(\sum_{s=0}^{\infty}\beta^s Z_s - \sum_{s=t+1}^{\infty}\beta^{s-(t+1)}Z_s + \frac{\beta}{1-\beta}ra_{-1}\right)$$

and therefore $\mathbf{A}^{TA} = (1-\mu)\mathbf{A}^{RA}$.

**BU model steady state.** In the steady state of the BU model, we have $a^u = a$ and $c^u = c$. Then, (A.56) implies

$$\beta(1+r) = 1 - \frac{\chi'(a)}{u'(c)} \tag{A.62}$$

since we have assumed that $u'(c^u) > 0$, we therefore

$$\beta(1+r) < 1 \qquad \Leftrightarrow \qquad \chi'(a) > 0$$

In other words, $\beta(1+r)$ can be on either side of 1, depending on $\chi'(a)$, as claimed in the main text.

Combining (A.56) and (A.57), we obtained an equation that implicitly defines the model's steady-state asset demand curve, $a(Z)$,

$$u'(Z+ra)(1-\beta(1+r)) = \chi'(a)$$

Starting from steady state and totally differentiating this equation, we obtain

$$\left(\frac{\chi''(a)}{u''(c)} - r(1-\beta(1+r))\right)da = (1-\beta(1+r))\,dZ$$

and substituting (A.62), we find

$$\left(\frac{\chi''(a)}{u''(c)} - r\frac{\chi'(a)}{u'(c)}\right)da = (1-\beta(1+r))\,dZ$$

We assume the stability condition:

$$\frac{\chi''(a)}{u''(c)} \geq r \frac{\chi'(a)}{u'(c)} \tag{A.63}$$

this ensures, in particular, that $\beta(1+r) < 1 \Leftrightarrow \frac{da}{dZ} > 0$.

**BU model Jacobians.** Totally differentiating the Euler equation (A.55) and the budget constraint (22) around the steady state $dZ_t$, we find

$$\begin{aligned} u''(c)\, dc_t &= \beta(1+r)\, u''(c)\, dc_{t+1} + \chi''(a)\, da_t \\ dc_t + da_t &= (1+r)\, da_{t-1} + dZ_t \end{aligned}$$

Combining, we obtain the second-order difference equation

$$\frac{1}{\beta} da_{t-1} - \left( \frac{1}{\beta(1+r)} + \frac{1}{\beta(1+r)} \frac{\chi''(a)}{u''(c)} + (1+r) \right) da_t + da_{t+1} = dZ_{t+1} - \frac{1}{\beta(1+r)} dZ_t \tag{A.64}$$

which characterizes the dynamics of this system. Consider the roots of the polynomial

$$P(X) = \frac{1}{\beta} - \left( \frac{1}{\beta(1+r)} + \frac{1}{\beta(1+r)} \frac{\chi''(a)}{u''(c)} + (1+r) \right) X + X^2 \tag{A.65}$$

We have $P(0) = \frac{1}{\beta} > 0$ and

$$P(1) = -\frac{1}{\beta(1+r)} \frac{\chi''(a)}{u''(c)} + r \left( \frac{1}{\beta(1+r)} - 1 \right)$$

this is negative since $\frac{\chi''(a)}{u''(c)} - r(1-\beta(1+r)) = \frac{\chi''(a)}{u''(c)} - r\frac{\chi'(a)}{u'(c)} \geq 0$ by the stability condition (A.63). Hence, there is a single root $0 < \lambda \leq 1$, and the other root is $\frac{1}{\beta\lambda} > 1$. By standard results, this implies that the solution to the difference equation (A.64) is:

$$da_t = \lambda da_{t-1} + \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} \left\{ \left( \frac{1}{\beta(1+r)} \right) dZ_{t+s} - dZ_{t+s+1} \right\}$$

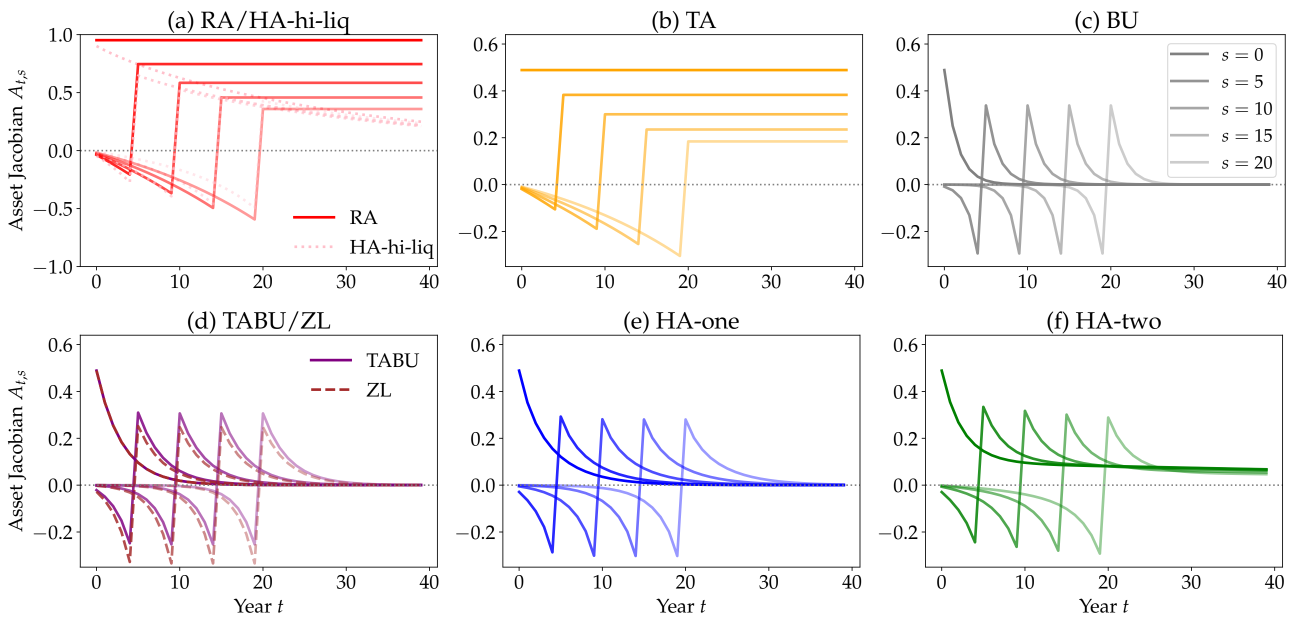
which we can reorganize as

$$da_t = \lambda da_{t-1} + \frac{\lambda}{1+r} dZ_t - \left( 1 - \frac{\lambda}{1+r} \right) \sum_{s=0}^{\infty} (\beta\lambda)^s dZ_{t+s} \tag{A.66}$$

This shows that the three parameters $\beta$, $r$ and $\lambda$ fully characterize the model's response to shocks. The root $\lambda$ represents the rate of decay of assets. Note that in the limit case where $\chi' = \chi'' = 0$, we have $\beta(1+r) = 1$ and $P(1) = 0$, so $\lambda = 1$ and we recover the RA model.

Using equation (A.66), we see that the asset Jacobian, $A_{ts} = \frac{\partial a_t}{\partial Z_s}$, is given by:

Figure D.1: **A** matrix for the eight standard models from figure 3




*Notes*: All models are calibrated to match $r = 0.05$. RA does not have any other free parameter. The single free parameter in BU, TA, HA-one and HA-two is calibrated to match $M_{00} = 0.51$. The additional free parameter in TABU and ZL is calibrated to match $M_{10} = 0.16$ (its value in the HA-one model).

$$\mathbf{A}^{BU} = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \frac{\lambda}{1+r} & -\left(1-\frac{\lambda}{1+r}\right)\cdot\beta\lambda & -\left(1-\frac{\lambda}{1+r}\right)\cdot(\beta\lambda)^2 & \cdots \\ 0 & \frac{\lambda}{1+r} & -\left(1-\frac{\lambda}{1+r}\right)\cdot\beta\lambda & \cdots \\ 0 & 0 & \frac{\lambda}{1+r} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{A.67}$$

The interpretation is as follows. When income is received immediately, agents save a fraction $1 - m = \frac{\lambda}{1+r}$, and then spend down this fraction at rate $\lambda$. When income is expected to be received $s$ periods in the future, agents dissave a fraction $m\,(\beta\lambda)^s$ in anticipation of that income. In addition, any past dissaving reduces current assets and therefore current consumption as agents aim to return their asset position to steady state at rate $\lambda$. The matrix product in (A.67) gives an analytical expression for the resulting asset dynamics.

Finally, applying equation (A.19), we obtain the **M** matrix of the BU model,

$$\mathbf{M}^{BU} = \mathbf{I} - (\mathbf{I} - (1+r)\,\mathbf{L})\mathbf{A}^{BU} \tag{A.68}$$

Manipulating this expression, we find that the first column of $\mathbf{M}^{BU}$ is $M_{t0}^{BU} = m \cdot \lambda^t$, where $m \equiv 1 - \frac{\lambda}{1+r}$ is the MPC. This can also be obtained directly from (A.66). First, when the shock is only to $dZ_0$, we have for $t = 0$ that $dc_0 = dZ_0 - da_0 = \left(1 - \frac{\lambda}{1+r}\right) dZ_0 = mdZ_0$, and for $t > 0$ that $dc_t = (1+r)\,da_{t-1} - da_t = \left(\frac{1+r}{\lambda} - 1\right) da_t = \left(1 - \frac{\lambda}{1+r}\right)\lambda^t = m\lambda^t$. We also find that the first row of

$\mathbf{M}^{BU}$ is $M^{BU}_{0s} = m \cdot (\beta\lambda)^s$. Here, since for $s > 0$ we have that $dc_0 = -da_0 = m (\beta\lambda)^s$.

This discussion makes clear that, given $r$, we can calibrate the model's $\lambda$ to hit a certain target for $M_{00} = m$, by taking $\lambda = (1+r)(1-m)$.

**Representative-agent special case.** In the representative-agent model, we have $\chi'(a) = \chi''(a) = 0$. By (A.62), we have $\beta(1+r) = 1$. The polynomial $P(X)$ in (A.65) is then $P(X) = \frac{1}{\beta} - \left(1+\frac{1}{\beta}\right)X + X^2$, with roots $\lambda = 1$ and $\frac{1}{\beta} > 1$. Plugging $\frac{1}{1+r} = \beta$ and $\lambda = 1$ into (A.67), we recover (A.60).

**Alternative derivation via policy functions.** We now derive the asset Jacobian (A.67) of the BU model through an alternative route, which will be helpful in connecting to our other results below. The value function of the BU model given a path of post-tax income $\{Z_s\}_{s\geq 0}$ is

$$
\begin{aligned}
V_t(a_-) &= \max_{c,a} u(c) + \chi(a) + \beta V_{t+1}(a) \\
& \quad c + a = Z_t + (1+r)a_-
\end{aligned}
\tag{A.69}
$$

Denote by $c_t(a_-)$, $a_t(a_-)$ the resulting policy functions. In a steady state with constant $Z$, let $c(a_-), a(a_-)$ denote the steady-state policy functions, $\overline{a}$ the steady-state level of assets (satisfying $\overline{a} = a(\overline{a})$), and $\lambda \equiv a'(\overline{a})$ denote the slope of the steady-state asset policy at $\overline{a}$.

The budget constraint implies that, at all $t$ and for all states $a_-$,

$$
c_t(a_-) + a_t(a_-) = Z_t + (1+r)a_- \tag{A.70}
$$

Equation (A.70) implies, in particular, that $c'(\overline{a}) + \lambda = 1+r$, and that, given any shock sequence $\{dZ_s\}$, we have also $dc_t(\overline{a}) + da_t(\overline{a}) = dZ_t$.

In this model, the distribution $\mu_t$ has a point mass at yesterday's aggregate assets $A_{t-1}$. Hence, aggregate assets are $A_t = a_t(A_{t-1})$ and consumption is $C_t = c_t(A_{t-1})$. Differentiating these relations, we obtain:

$$
dA_t = da_t(\overline{a}) + a'(\overline{a})\,dA_{t-1} = dZ_t - dc_t(\overline{a}) + \lambda dA_{t-1} \tag{A.71}
$$

$$
dC_t = dc_t(\overline{a}) + c'(\overline{a})\,dA_{t-1} = dc_t(\overline{a}) + (1+r-\lambda)\,dA_{t-1} \tag{A.72}
$$

To obtain aggregate dynamics, it remains to determine the change in policy $dc_t(\overline{a})$. To do this, we consider the effects of each shock $\{dZ_s\}$ on the policy in isolation, and then use linearity of the total derivative to add up the effects. For $s < t$, the policy is unaffected. For $s = t$, the value function $V_{t+1}$ is the steady-state $V$, and it follows from (A.70) that $dc_t(\overline{a}) = m dZ_t$, where $m = \frac{c'(\overline{a})}{1+r} = 1 - \frac{\lambda}{1+r}$. Finally, for $s > t$, the envelope condition $V'_t(a_-) = (1+r)u'(c_t(a_-))$ implies

$$
dV'_t(\overline{a}) = (1+r)u''(\overline{c})dc_t(\overline{a}) \tag{A.73}
$$

Since we can rewrite (A.69) in the form (A.51), with $F(a) \equiv u(Z + (1+r)a_- - a) + \chi(a)$ a constant function, we can apply the result from (A.53), which gives us $dV_t'(\bar{a}) = \beta\lambda dV_{t+1}'(\bar{a})$. Combining with (A.73), we obtain $dc_t(\bar{a}) = \beta\lambda dc_{t+1}(\bar{a})$ whenever the shock $dZ_s$ is at $s > t$. To conclude, we have, for an individual shock to $dZ_s$,

$$dc_t(\bar{a}) = \begin{cases} m(\beta\lambda)^{s-t} dZ_s & s \geq t \\ 0 & s < t \end{cases}$$

and adding up across all shocks $\{dZ_s\}_{s=0}^{\infty}$, this implies

$$dc_t(\bar{a}) = m\sum_{s=t}^{\infty}(\beta\lambda)^{s-t} dZ_s \tag{A.74}$$

Equations (A.71) and (A.74) can be immediately combined to give the asset Jacobian $\mathbf{A}$. Recursing on (A.71), we have:

$$dA_t = \sum_{u=0}^{t} \lambda^{t-u}(dZ_u - dc_u(\bar{a})) \tag{A.75}$$

while (A.74) implies

$$da_t(\bar{a}) = dZ_t - dc_t(\bar{a}) = (1-m)dZ_t - m\sum_{s=t+1}^{\infty}(\beta\lambda)^{s-t}dZ_s \tag{A.76}$$

In matrix form, combining (A.75) and (A.76), we obtain:

$$d\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} 1-m & -m(\beta\lambda) & -m(\beta\lambda)^2 & \cdots \\ 0 & 1-m & -m(\beta\lambda) & \cdots \\ 0 & 0 & 1-m & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} d\mathbf{Z} \tag{A.77}$$

where, as shown above, $m = 1 - \frac{\lambda}{1+r}$. This gives the same equation for $\mathbf{A}$ as (A.67). Finally, to obtain the $\mathbf{M}$ matrix, we use (A.72), which can for instance be expressed as $dC_t = (1+r)dA_{t-1} - dA_t + dZ_t$, and give equation (A.68).

Note that this derivation does not directly give us the root $\lambda$. To obtain this, we use the functional Euler equation

$$u'(c(a_-)) = \chi'(a(a_-)) + \beta(1+r)u'(c(a(a_-)))$$

differentiating and evaluating at the steady-state level $\bar{a}, \bar{c}$, we find

$$u''(\bar{c})(1+r-\lambda) = \chi''(\bar{a})\lambda + \beta(1+r)u''(\bar{c})(1+r-\lambda)\lambda$$

which can be verified to give the equation $P(\lambda) = 0$ where $P$ is defined in (A.65).

**TABU model.** The TABU model has

$$\mathbf{M}^{TABU} = (1-\mu)\,\mathbf{M}^{BU} + \mu\mathbf{I} = \mathbf{I} - (1-\mu)\,(\mathbf{I} - (1+r)\,\mathbf{L})\mathbf{A}^{BU} \tag{A.78}$$

therefore,

$$\mathbf{A}^{TABU} = (1-\mu)\,\mathbf{A}^{BU} \tag{A.79}$$

Hence, this model has four parameters, $\beta, r, \lambda$ and $\mu$. Given $\beta, r$, we can therefore solve for $\mu, \lambda$ in terms of $M_{00}$ and $M_{10}$, with $M_{00} = \mu + (1-\mu)\left(1 - \frac{\lambda}{1+r}\right)$ and $M_{10} = (1-\mu)\left(1-\frac{\lambda}{1+r}\right)\lambda$.

**Quasi-Toeplitz structure of $\mathbf{M}^{TABU}$ and $\mathbf{A}^{TABU}$.** Putting together (A.79), and (A.67), we see that $\mathbf{A}^{TABU}$ is the product of two Toeplitz matrices, $\mathbf{A}^{TABU} = \mathbf{T}(a_+)\,\mathbf{T}(a_-)$. By standard results (eg Böttcher and Grudsky 2005), $\mathbf{A}^{TABU}$ is therefore the sum of the Toeplitz matrix $\mathbf{T}(a_- a_+)$ and a compact matrix $\mathbf{E}$ which satisfies $E_{t,s} \to 0$ as $t, s \to \infty$. Hence, $A^{TABU}_{t,s} \to a^{TABU}_{t-s}$ for $t, s \to \infty$. We say that $\mathbf{A}^{TABU}$ is *quasi-Toeplitz*: as shown in Auclert et al. (2023b), this is a general property of the Jacobians of stationary heterogeneous-agent models. Since we have an exact analytical expression for the $\mathbf{A}$ of the TABU model, we now directly calculate its asymptotic Toeplitz form.

From (A.67) and (A.79), we can directly calculate the symbols:

$$a_+(z) = \frac{1-\mu}{1-\lambda z} \tag{A.80}$$

$$a_-(z) = 1 - \frac{1-\frac{\lambda}{1+r}}{1-\beta\lambda z^{-1}} = \frac{\frac{\lambda}{1+r} - \beta\lambda z^{-1}}{1-\beta\lambda z^{-1}} = \frac{\lambda}{1+r}\frac{1-\beta(1+r)z^{-1}}{1-\beta\lambda z^{-1}} \tag{A.81}$$

and decomposing $a^{TABU}(z) = a_+(z)\,a_-(z)$ into partial fractions, we find

$$\begin{aligned} a^{TABU}(z) &= (1-\mu)\frac{\lambda}{1+r}\frac{1}{1-\beta\lambda^2}\left(\frac{1-\beta\lambda(1+r)}{1-\lambda z} - \frac{\beta(1+r)\left(1-\frac{\lambda}{1+r}\right)}{1-\beta\lambda z^{-1}}z^{-1}\right) \\ &= \frac{1-\mu}{1-\beta\lambda^2}\left(\frac{\lambda}{1+r}(1-\beta\lambda(1+r))\sum_{k=0}^{\infty}\lambda^k z^k - \left(1-\frac{\lambda}{1+r}\right)\sum_{k=1}^{\infty}(\beta\lambda)^k z^{-k}\right) \end{aligned}$$

Hence, the asymptotic Toeplitz column for $\mathbf{A}^{TABU}$ has a simple double-exponential form with rate of decay $\lambda$ on the right and $\beta\lambda$ on the left:

$$a_k^{TABU} = \frac{1-\mu}{1-\beta\lambda^2}\begin{cases} -\left(1-\frac{\lambda}{1+r}\right)(\beta\lambda)^{-k} & k<0 \\ \frac{\lambda}{1+r}(1-\beta\lambda(1+r))\,\lambda^k & k \geq 0 \end{cases} \tag{A.82}$$

Similarly, $\mathbf{M}^{TABU}$ is the sum of products of Toeplitz matrices, so satisfies $M^{TABU}_{t,s} \to m^{TABU}_{t-s}$ for

$t,s\to\infty$, and we can obtain:

$$\begin{aligned} m^{TABU}(z) &= \mu+(1-\mu)\left(1-(1-(1+r)z)\frac{a^{TABU}(z)}{1-\mu}\right) \\ &= \mu+(1-\mu)\frac{\left(1-\frac{\lambda}{1+r}\right)(1-\beta\lambda(1+r))}{(1-\lambda z)(1-\beta\lambda z^{-1})} \end{aligned}$$

again decomposing $m^{TABU}(z)$ into partial fractions, we find

$$\begin{aligned} m^{TABU}(z) &= \mu+(1-\mu)\left(1-\frac{\lambda}{1+r}\right)(1-\beta\lambda(1+r))\frac{1}{1-\beta\lambda^2}\left(\frac{1}{1-\lambda z}+\frac{\beta\lambda z^{-1}}{1-\beta\lambda z^{-1}}\right) \\ &= \mu+(1-\mu)\left(1-\frac{\lambda}{1+r}\right)(1-\beta\lambda(1+r))\frac{1}{1-\beta\lambda^2}\left(\sum_{k=0}^{\infty}\lambda^k z^k+\sum_{k=1}^{\infty}(\beta\lambda)^k z^{-k}\right) \end{aligned}$$

Hence, the asymptotic Toeplitz column for $\mathbf{M}^{TABU}$ also has the simple double-exponential form:

$$m_k^{TABU}=\mu 1_{k=0}+(1-\mu)\left(1-\frac{\lambda}{1+r}\right)\frac{1-\beta\lambda(1+r)}{1-\beta\lambda^2}\begin{cases}(\beta\lambda)^{-k} & k<0\\ \lambda^k & k\geq 0\end{cases} \tag{A.83}$$

Equation (A.83) that, in the limit, the rate of decay of consumption after income is received is exactly $\lambda$, and the rate of anticipation of future income is exactly $\beta\lambda$.

This limit is attained quickly in practice, as is visible in figures 3 and D.1 for $s\simeq 10$.

### D.3 One-account HA model

The one-account model is the special case of the model of section A.1 where assets can only be invested in a single, liquid account. Defining $\varepsilon\equiv\frac{e^{1-\theta}}{E[e^{1-\theta}]}$ for simplicity ($\varepsilon$ follows a Markov process with the same Markov transition matrix as $e$), and $r_t$ for the interest on the bond between time $t$ and time $t+1$, the Bellman equation for this model is:

$$\begin{aligned} V_t(\varepsilon,a_-) &= \max_{c,a} u(c)+\beta\mathbb{E}\left[V_{t+1}(\varepsilon',a)\,|\varepsilon\right] \\ & c+a=\varepsilon Z_t+(1+r_{t-1})a_- \\ & a\geq 0 \end{aligned} \tag{A.84}$$

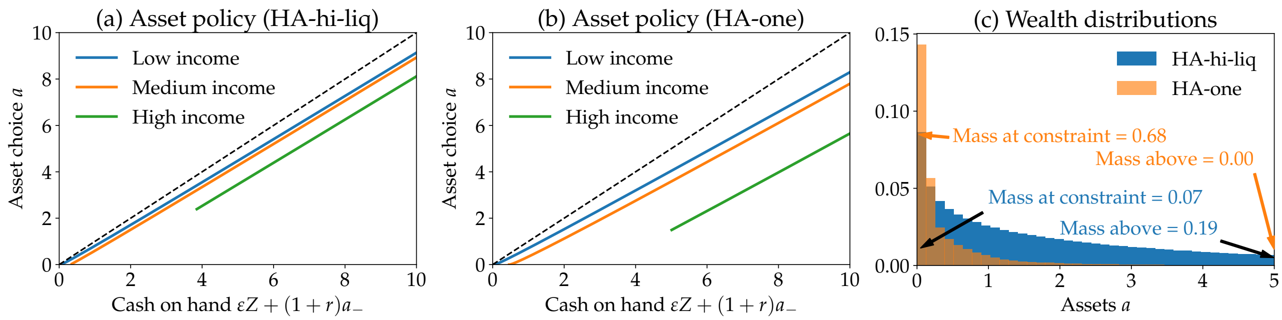
The first-order condition for this problem is:

$$u'(c_t(\varepsilon,a_-)) \geq \beta\sum_{\varepsilon'}\Pi_{\varepsilon\varepsilon'}\frac{\partial V_{t+1}}{\partial a_-}(\varepsilon',a') \tag{A.85}$$

where a strict inequality implies a binding borrowing constraint $c_t(\varepsilon,a_-)=\varepsilon Z_t+(1+r)a_-$. The envelope condition is:

$$\frac{\partial V_t}{\partial a_-}(\varepsilon,a_-)=(1+r_{t-1})u'(c_t(\varepsilon,a_-)) \tag{A.86}$$

Figure D.2: Steady-state policies and distribution in HA-hi-liq and HA-one




Note: Panels (a) and (b) report the asset policy $a(\varepsilon, a_-)$ as a function of the level of cash-on-hand $\varepsilon Z + (1+r)a_-$. Calibration parameters are in Table 2, and distributional statistics are in Table D.1. A low income consumer has $\epsilon = 0.07$, a medium income consumer has $\epsilon = 0.75$, and a high income consumer has $\epsilon = 8.2$. Panel (c) reports the wealth distributions in these two calibrations for $a \in (0, 5)$, as well as the mass below and above these two end points.

Combining these two conditions, we get the standard Euler equation

$$u'(c_t(\varepsilon, a_-)) \geq \beta(1+r_t)\mathbb{E}\left[u'\left(c_{t+1}\left(\varepsilon', a\right)\right)|\epsilon\right] \tag{A.87}$$

We discretize the AR(1) process for $\mathbf{e}$ on an 11 point grid, using the Rouwenhorst method, from which we obtain $\varepsilon$ by rescaling the grid. We solve for $\frac{\partial V_t}{\partial a_-}(\varepsilon, a_-)$ and $c_t(\varepsilon, a)$ on this grid for $\varepsilon$ and 200-point, double-exponentially-spaced grid for assets $a$; using the method of endogenous grid points to obtain $\frac{\partial V_t}{\partial a_-}(\varepsilon, a_-)$ from $\frac{\partial V_{t+1}}{\partial a_-}(\varepsilon, a_-)$. We then solve for the dynamics of the distribution $D_t(\epsilon, a)$ using non-stochastic simulation. Aggregating policies with distributions, we obtain the consumption and asset functions $C_t(\{Z_s\})$ and $A_t(\{Z_s\})$. To obtain the Jacobians of these functions $\mathbf{M}$ and $\mathbf{A}$, we build up from their "fake-news matrices", following the method in Auclert et al. (2021a).

As described in the main text, we consider two calibrations of this model. In our first calibration (HA-hi-liq), we pick $\beta$ is chosen to hit a target for aggregate assets $A = \int a_{it} di$, such that $A/Z$ has the same value 6.29 as in our quantitative model. Table 2 shows that this results in $\beta = 0.94$, just slightly below $1/(1+r)$. In our second calibration (HA-one), we pick $\beta$ to hit a target for $M_{00}$, such that $M_{00}$ is the same as the point estimate 0.51 from the Norwegian data. Table 2 shows that this results in a much lower $\beta = 0.87$, well below $1/(1+r)$.

Figure D.2, panels (a) and (b) display the steady-state asset policy functions in these two calibrations. We report the asset policy $a(\varepsilon, a_-)$ as a function of the level of cash-on-hand $\varepsilon Z + (1+r)a_-$, for a low income consumer ($\epsilon = 0.07$), medium income consumer ($\epsilon = 0.75$) and high income consumer ($\epsilon = 8.2$). Since $c + a = \varepsilon Z + (1+r)a_-$, the distance to the 45 degree line is the consumption policy. In the HA-hi-liq calibration, agents leave themselves with no assets, $a = 0$, only for very low values of cash on hand, and save aggressively for higher values of cash-on-hand. As a consequence, the stationary asset distribution only has 7% of agents at the borrowing constraint, and the distribution is quite spread out (see figure D.2, panel (c)). By contrast, in the HA-one calibration, agents are much more impatient: they leave themselves with no assets for a

| Parameter | Data | HA-hi-liq | HA-one | HA-two |
|---|---|---|---|---|
| Share of HtM ($a^{liq} = 0$) | | 0.04 | 0.58 | 0.58 |
| Share with liquid assets $a^{liq} \leq 0.1$ | | 0.14 | 0.80 | 0.78 |
| Share of WHtM ($a^{liq} = 0$, $a^{illiq} > 0$) | | — | — | 0.28 |
| Share of income accruing to HtM | | 0.01 | 0.30 | 0.36 |
| Share of income accruing to $a^{liq} \leq 0.1$ | | 0.05 | 0.57 | 0.61 |
| Share of income accruing to WHtM | | — | — | 0.22 |
| Gini for total wealth | 0.83 | 0.60 | 0.90 | 0.94 |
| Top 10% wealth share | 0.71 | 0.40 | 0.83 | 0.89 |
| Aggregate post-tax income $Z$ | | 0.47 | 0.61 | 0.47 |

Note: Source for data: World Inequality Database, `https://wid.world/country/usa/`. U.S. values are for 2021.

Table D.1: Distributional statistics for calibrated models

much broader range of values of cash on hand, and the slope of the asset policy is much lower. As a consequence, the stationary asset distribution is much more concentrated around 0, with 67% of agents at the constraint.[A-25]

Table D.1 reports distributional statistics for these two calibrations. The share of income accruing to hand-to-mouth agents is 1% in the HA-hi-liq calibration and 30% in the HA-one calibration: these numbers are more relevant to understand iMPCs than the pure share of constrained agents, since our iMPCs are income-weighted. The Gini coefficient is 0.6 in HA-hi-liq and 0.90 in HA-one; the top 10% wealth shares are 40% and 83% respectively. These numbers lie between the corresponding numbers in the U.S. data (source: World Inequality Database, 2021 number).

We construct two pairs of Jacobians $(\mathbf{M}, \mathbf{A})$ for our two calibrations. We report columns of these Jacobians in figures 2 and D.1, panel (a) for HA-hi-liq and panel (d) for HA-one.

## D.4 Zero liquidity model and relation to TABU

Suppose that we start from a calibration with given $(\Pi, \mathbf{e}, r)$ and then keep recalibrating the model by changing $\beta$ as we take the limit $A \to 0$. In this "zero-liquidity limit" steady state, households consume exactly their after-tax labor incomes. Equilibrium $r$ is such that the intertemporal Euler equation holds with equality for whatever income state $\bar{e}$ has the highest incentive to save, i.e. $1 + r = \min_{\bar{e}} \frac{1}{\beta} u'\left(e^{1-\theta}\right) / \sum_e \Pi_{\bar{e}e} u'\left(e^{1-\theta}\right)$, where we assume that $\bar{e}$ is unique. In every other income state $e \neq \bar{e}$, the borrowing constraint is strictly binding, as agents would like to borrow but cannot.

This zero-liquidity limit—which we will abbreviate as the *ZL* model—has been widely applied in the literature, since the degenerate asset distribution at zero radically simplifies the steady state and makes the model more analytically tractable. Here, we show that this tractability extends to intertemporal MPCs as well.

[A-25] Table D.1 reports the share of agents who choose $a = 0$ for the following period in these two calibrations. The number is 4% and 58% for HA-one. These are close to, but not equal, to the number of agents with $a = 0$ in the stationary distribution, since some agents that pick assets close to 0 end up exactly at 0 after they are reallocated to the grid in our non-stochastic simulation.

**Derivation.** Let us define $\varepsilon \equiv \frac{e^{1-\theta}}{\mathbb{E}[e^{1-\theta}]}$. $\varepsilon$ follows a Markov process with the same Markov transition matrix as $e$. Substituting $c_{it} = \varepsilon_{it} Z_t$ into the first-order conditions (A.85)–(A.86) of the one-account HA model and using the CES form $u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$, we have that for all $\varepsilon$,

$$(\varepsilon)^{-\frac{1}{\sigma}} \geq \beta(1+r)\sum_{\varepsilon'} \Pi_{\varepsilon\varepsilon'}\left(\varepsilon' Z_{t+1}\right)^{-\frac{1}{\sigma}}$$

Defining $\rho(\varepsilon) \equiv \sum_{\varepsilon'} \Pi_{\varepsilon\varepsilon'}\left(\frac{\varepsilon'}{\varepsilon}\right)^{-\frac{1}{\sigma}}$, this equation reads $(Z_t)^{-\frac{1}{\sigma}} \geq \beta(1+r)\rho(\varepsilon)(Z_{t+1})^{-\frac{1}{\sigma}}$ for all $\varepsilon$. We assume that there is a unique $\bar{\varepsilon}$ achieving $\bar{\varepsilon} = \arg\max_{\varepsilon} \rho(\varepsilon)$. In the zero liquidity limit, we have at every $t$

$$(Z_t)^{-\frac{1}{\sigma}} = \beta(1+r)\rho(\bar{\varepsilon})(Z_{t+1})^{-\frac{1}{\sigma}}$$

and in particular, considering the steady state with $Z_t \equiv Z$, we have $\beta(1+r)\rho(\bar{\varepsilon}) = 1$. This equation determines the equilibrium $r$ in steady state.

**Policies.** We next derive the policy functions in the zero liquidity model. This section closely parallels the section "Alternative derivation via policy functions" for the BU model in section D.2.

Let $c_t(\varepsilon, a_-)$, $a_t(\varepsilon, a_-)$ the policy functions when the time path of income is $\{Z_t\}$, with $c(\varepsilon, a_-)$, $a(\varepsilon, a_-)$ the steady-state policies. We know that the steady state level of assets is $\bar{a} = 0$, with $c(\varepsilon, 0) = \varepsilon Z$ and $a(\varepsilon, 0) = 0$. Moreover, let $\lambda_\varepsilon \equiv \frac{\partial a(\varepsilon,0)}{\partial a_-}$ denote the slope of the asset policy in state $e$. From the budget constraint,

$$c_t(\varepsilon, a_-) + a_t(\varepsilon, a_-) = \varepsilon Z_t + (1+r)a_-$$

we have that $\frac{\partial c}{\partial a_-}(e, 0) + \lambda_\varepsilon = 1 + r$. Moreover, given any shock sequence $\{dZ_s\}$, we also have $dc_t(\varepsilon, 0) + da_t(\varepsilon, 0) = \varepsilon dZ_t$.

We again consider the effects of each shock $\{dZ_s\}$ on the policy in isolation, and then use linearity of the total derivative to add up the effects. For $s < t$, the policy is unaffected. For $s = t$, the value function $V_{t+1}$ is the steady state $V$, and it follows from (A.70) that $dc_t(\varepsilon, 0) = m_\varepsilon \varepsilon dZ_t$, where $m_\varepsilon \equiv \frac{1}{1+r}\frac{\partial c}{\partial a_-}(\varepsilon, 0) = 1 - \frac{\lambda_\varepsilon}{1+r}$ is the marginal propensity to consume of an agent in state $\varepsilon$ with zero assets. Moreover, for $\varepsilon \neq \bar{\varepsilon}$, the borrowing constraint is binding, $a_t(\varepsilon, a_-) = 0$, so it immediately follows $\lambda_\varepsilon = 0$ and $m_\varepsilon = 1$. In other words, the marginal propensity to consume of agents in all but the top state is 1.

Then, for $t < s$, we apply the result from section D.1, which here shows that that for all $\varepsilon$, we have

$$dV_t'(\varepsilon, 0) = \beta\lambda_\varepsilon \sum_{\varepsilon'} \Pi_{\varepsilon\varepsilon'} dV_{t+1}'(\varepsilon', 0)$$

Using (A.86), which defines $V_t'(\varepsilon, a_-) = (1+r)u'(c_t(\varepsilon, a_-))$, we have at all $t$ and $\varepsilon$,

$$dV_t'(\varepsilon, 0) = (1+r)u''(\varepsilon Z)dc_t(\varepsilon, 0)$$

and so, using the fact that $\frac{u''(\varepsilon' Z)}{u''(\varepsilon Z)} = \left(\frac{\varepsilon'}{\varepsilon}\right)^{-\left(\frac{1}{\sigma}+1\right)}$, we obtain for every $t$ and $\varepsilon$:

$$dc_t(\varepsilon, 0) = \beta\lambda_\varepsilon \sum_{\varepsilon'} \Pi_{\varepsilon\varepsilon'} \left(\frac{\varepsilon'}{\varepsilon}\right)^{-\left(\frac{1}{\sigma}+1\right)} dc_{t+1}(\varepsilon', 0) \tag{A.88}$$

Given that $\lambda_\varepsilon = 0$ for all $\varepsilon \neq \bar{\varepsilon}$, we can solve equation (A.88) forward. When the shock is at $s = t+1$, we have $dc_t(\bar{\varepsilon}, 0) = \beta\lambda_{\bar{\varepsilon}} \left( \sum_{\varepsilon' \neq \bar{\varepsilon}} \Pi_{\bar{\varepsilon}\varepsilon'} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\left(\frac{1}{\sigma}+1\right)} \cdot \varepsilon + \Pi_{\bar{\varepsilon}\bar{\varepsilon}} \cdot m_{\bar{\varepsilon}} \bar{\varepsilon} \right) dZ_{t+1}$, and when is the shock is at $s > t+1$, we have $dc_t(\bar{\varepsilon}, 0) = (\beta\lambda_{\bar{\varepsilon}}\Pi_{\bar{\varepsilon}\bar{\varepsilon}}) dc_{t+1}(\bar{\varepsilon}, 0)$. In summary, we have:

$$dc_t(\varepsilon, 0) = \begin{cases} (\beta\lambda_{\bar{\varepsilon}}\Pi_{\bar{\varepsilon}\bar{\varepsilon}})^{s-t} \left( \sum_{\varepsilon' \neq \bar{\varepsilon}} \frac{\Pi_{\bar{\varepsilon}\varepsilon'}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\left(\frac{1}{\sigma}+1\right)} \cdot \varepsilon' + m_{\bar{\varepsilon}}\bar{\varepsilon} \right) dZ_s & s > t, \varepsilon = \bar{\varepsilon} \\ m_\varepsilon \varepsilon dZ_t & s = t, \varepsilon \neq \bar{\varepsilon} \\ 0 & s < t \end{cases}$$

and adding up across all shocks $\{dZ_s\}_{s=0}^\infty$, this implies $dc_t(\varepsilon, 0) = \varepsilon dZ_t$ for all $\varepsilon \neq \bar{\varepsilon}$, and for $\varepsilon = \bar{\varepsilon}$,

$$dc_t(\bar{\varepsilon}, 0) = m_{\bar{\varepsilon}}\bar{\varepsilon} dZ_t + \left( \sum_{\varepsilon' \neq \bar{\varepsilon}} \frac{\Pi_{\bar{\varepsilon}\varepsilon'}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\left(\frac{1}{\sigma}+1\right)} \cdot \varepsilon' + m_{\bar{\varepsilon}}\bar{\varepsilon} \right) \sum_{s>t}^{\infty} (\beta\lambda_{\bar{\varepsilon}}\Pi_{\bar{\varepsilon}\bar{\varepsilon}})^{s-t} dZ_s$$

Combining with $dc_t(\bar{\varepsilon}, 0) + da_t(\bar{\varepsilon}, 0) = \bar{\varepsilon} dZ_t$, we finally obtain: $da_t(\varepsilon, 0) = 0$ for all all $\varepsilon \neq \bar{\varepsilon}$, and for $\varepsilon = \bar{\varepsilon}$,

$$da_t(\bar{\varepsilon}, 0) = \bar{\varepsilon}(1 - m_{\bar{\varepsilon}}) dZ_t - \bar{\varepsilon} \left( \sum_{e' \neq \bar{e}} \frac{\Pi_{\bar{\varepsilon}\varepsilon'}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\frac{1}{\sigma}} + m_{\bar{\varepsilon}} \right) \sum_{s>t}^{\infty} (\beta\lambda_{\bar{\varepsilon}}\Pi_{\bar{\varepsilon}\bar{\varepsilon}})^{s-t} dZ_s \tag{A.89}$$

**Aggregation.** Since the borrowing constraint is binding, $a_t(\varepsilon, a_-) = 0$, for all agents except those with $\varepsilon = \bar{\varepsilon}$, the only agents that ever have any assets after a shock are those in state $\bar{\varepsilon}$. Moreover, all agents that have spent the same number $i$ of continuous periods in state $\bar{\varepsilon}$ have the same behavior, so they have the same level of assets $a_{it}$, described by:

$$a_{it} = a_t(\bar{\varepsilon}, a_{i-1,t-1}) \quad \forall t \geq 1, i \geq 0 \tag{A.90}$$

where we define $a_{-1,t} = 0$. At each $t$, there is a fraction $\pi_{\bar{\varepsilon}}(1 - \Pi_{\bar{\varepsilon}\bar{\varepsilon}}) \Pi_{\bar{\varepsilon}\bar{\varepsilon}}^i$ of agents in state $i$, where $\pi_{\bar{\varepsilon}}$ is the number of agents in sate $\bar{\varepsilon}$ overall and $\Pi_{\bar{\varepsilon}\bar{\varepsilon}}$ the probability that agents stay in that state. Aggregate assets are therefore

$$A_t = \sum_{i=0}^{\infty} \pi_{\bar{\varepsilon}}(1 - \Pi_{\bar{\varepsilon}\bar{\varepsilon}}) \Pi_{\bar{\varepsilon}\bar{\varepsilon}}^i a_{it} \tag{A.91}$$

Differentiating (A.90) around the steady state and using $\lambda_{\bar{\varepsilon}} = \frac{\partial a_t}{\partial a_-}(\bar{\varepsilon}, 0)$, we have, for all $t \geq 1, i \geq 0$,

$$da_{i,t} = da_t(\bar{\varepsilon}, 0) + \lambda_{\bar{\varepsilon}} da_{i-1,t-1}$$

where $da_{-1,t} = 0$. Aggregating with weights $\pi_{\bar{\varepsilon}} (1 - \Pi_{\bar{\varepsilon}\bar{\varepsilon}}) \Pi^i_{\bar{\varepsilon}\bar{\varepsilon}}$ and using (A.91) the left-hand side is $dA_t$. For the left-hand side, we using the fact that $\sum_{i=0}^{\infty} \pi_{\bar{\varepsilon}} (1 - \Pi_{\bar{\varepsilon}\bar{\varepsilon}}) \Pi^i_{\bar{\varepsilon}\bar{\varepsilon}} = \pi_{\bar{\varepsilon}}$ and, given $da_{-1,t-1} = 0$,

$$\sum_{i=0}^{\infty} \pi_{\bar{\varepsilon}} (1 - \Pi_{\bar{\varepsilon}\bar{\varepsilon}}) \Pi^i_{\bar{\varepsilon}\bar{\varepsilon}} da_{i-1,t-1} = \Pi_{\bar{\varepsilon}\bar{\varepsilon}} \sum_{i=0}^{\infty} \pi_{\bar{\varepsilon}} (1 - \Pi_{\bar{\varepsilon}\bar{\varepsilon}}) \Pi^i_{\bar{\varepsilon}\bar{\varepsilon}} da_{i,t-1} = \Pi_{\bar{\varepsilon}\bar{\varepsilon}} dA_{t-1}$$

Hence, we obtain the law of motion for aggregate assets:

$$dA_t \quad = \quad da_t (0, \bar{\varepsilon}) \pi_{\bar{\varepsilon}} + \lambda_{\bar{\varepsilon}} \Pi_{\bar{\varepsilon}\bar{\varepsilon}} dA_{t-1} \tag{A.92}$$

The asset Jacobian follows from combining (A.89) with (A.92). Let $\lambda \equiv \lambda_{\bar{\varepsilon}} \Pi_{\bar{\varepsilon}\bar{\varepsilon}}$. Since $m_{\bar{\varepsilon}} = 1 - \frac{\lambda_{\bar{\varepsilon}}}{1+r}$, we have $1 - m_{\bar{\varepsilon}} = \frac{\lambda_{\bar{\varepsilon}}}{1+r} = \frac{1}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \frac{\lambda}{1+r}$ . This implies:

$$\mathbf{A}^{ZL} = \frac{\pi_{\bar{\varepsilon}} \bar{\varepsilon}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \frac{\lambda}{1+r} & -\Pi_{\bar{\varepsilon}\bar{\varepsilon}} (m_{\bar{\varepsilon}} + K) (\beta\lambda) & -\Pi_{\bar{\varepsilon}\bar{\varepsilon}} (m_{\bar{\varepsilon}} + K) (\beta\lambda)^2 & \cdots \\ 0 & \frac{\lambda}{1+r} & -\Pi_{\bar{\varepsilon}\bar{\varepsilon}} (m_{\bar{\varepsilon}} + K) (\beta\lambda) & \cdots \\ 0 & 0 & \frac{\lambda}{1+r} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where $K \equiv \sum_{\varepsilon' \neq \bar{\varepsilon}} \frac{\Pi_{\bar{\varepsilon}\varepsilon'}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\frac{1}{\sigma}}$. But

$$\begin{aligned} \Pi_{\bar{\varepsilon}\bar{\varepsilon}} (m_{\bar{\varepsilon}} + K) &= \Pi_{\bar{\varepsilon}\bar{\varepsilon}} - \frac{\lambda}{1+r} + \Pi_{\bar{\varepsilon}\bar{\varepsilon}} K \\ &= \Pi_{\bar{\varepsilon}\bar{\varepsilon}} (K+1) - \frac{\lambda}{1+r} \end{aligned}$$

Now, since $K = \sum_{\varepsilon' \neq \bar{\varepsilon}} \frac{\Pi_{\bar{\varepsilon}\varepsilon'}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\frac{1}{\sigma}}$, we have $\Pi_{\bar{\varepsilon}\bar{\varepsilon}} (K+1) = \sum_{\varepsilon'} \Pi_{\bar{\varepsilon}\varepsilon'} \left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\frac{1}{\sigma}} = \rho(\bar{\varepsilon}) = \frac{1}{\beta(1+r)}$ at steady state. So, we have simply

$$\Pi_{\bar{\varepsilon}\bar{\varepsilon}} (m_{\bar{e}} + K) = \frac{1}{\beta(1+r)} - \frac{\lambda}{1+r}$$

We conclude that the asset Jacobian of the ZL model is

$$\mathbf{A}^{ZL} = (1-\mu) \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \frac{\lambda}{1+r} & -\left(\frac{1}{\beta(1+r)} - \frac{\lambda}{1+r}\right)(\beta\lambda) & -\left(\frac{1}{\beta(1+r)} - \frac{\lambda}{1+r}\right)(\beta\lambda)^2 & \cdots \\ 0 & \frac{\lambda}{1+r} & -\left(\frac{1}{\beta(1+r)} - \frac{\lambda}{1+r}\right)(\beta\lambda) & \cdots \\ 0 & 0 & \frac{\lambda}{1+r} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{A.93}$$

where the effective hand-to-mouth share is $1 - \mu \equiv \frac{\pi_{\bar{\varepsilon}} \bar{\varepsilon}}{\Pi_{\bar{\varepsilon}\bar{\varepsilon}}} = \frac{\pi_{\bar{e}}}{\Pi_{\bar{e}\bar{e}}} \frac{e^{1-\theta}}{\mathbb{E}[e^{1-\theta}]}$. This has almost the same form as (A.79), except for the fact that there is a factor $\frac{1}{\beta(1+r)} - \frac{\lambda}{1+r} > 1 - \frac{\lambda}{1+r}$ governing anticipation.

**Deriving the rate of decay of assets $\lambda$ from primitives.** We finally solve for $m_{\bar{\varepsilon}}$, which allows us to obtain $\lambda$ from primitives. Consider the Euler equation at steady state, for the agent $\bar{\varepsilon}$,

$$c\left(\bar{\varepsilon}, a_{-}\right)^{-\frac{1}{\sigma}} = \beta\left(1+r\right)\sum_{\varepsilon'}\Pi_{\bar{\varepsilon}\varepsilon'}\left(c\left(\varepsilon', a\left(\bar{\varepsilon}, a_{-}\right)\right)\right)^{-\frac{1}{\sigma}}$$

We have $m_{\bar{\varepsilon}} \equiv \frac{1}{1+r}\frac{\partial c}{\partial a_{-}}\left(\bar{\varepsilon}, 0\right) = 1 - \frac{\lambda_{\bar{\varepsilon}}}{1+r}$. Differentiating the above with respect to $a_{-}$ and evaluating at $a_{-} = 0$, we have:

$$u''\left(\bar{\varepsilon}\right)\frac{\partial c}{\partial a_{-}}\left(\bar{\varepsilon}, 0\right) = \beta\left(1+r\right)\sum_{\varepsilon'}\Pi_{\bar{\varepsilon}\varepsilon'}u''\left(\varepsilon'\right)\frac{\partial c}{\partial a_{-}}\left(\varepsilon', 0\right)\cdot\frac{\partial a}{\partial a_{-}}\left(\bar{\varepsilon}, 0\right)$$

but, by definition, $\frac{\partial c}{\partial a_{-}}\left(\varepsilon, 0\right) = \left(1+r\right)m_{\varepsilon}$, and also $\frac{\partial a}{\partial a_{-}}\left(\bar{\varepsilon}, 0\right) = \left(1+r\right)\left(1-m_{\bar{\varepsilon}}\right)$.This implies:

$$u''\left(\bar{\varepsilon}\right)m_{\bar{\varepsilon}} = \beta\left(1+r\right)^{2}\sum_{\varepsilon'}\Pi_{\bar{\varepsilon}\varepsilon'}u''\left(\varepsilon'\right)m_{\varepsilon'}\cdot\left(1-m_{\bar{\varepsilon}}\right)$$

Using the functional form for $u$, $\frac{u''(\varepsilon')}{u''(\bar{\varepsilon})} = \left(\frac{\varepsilon'}{\varepsilon}\right)^{-\left(\frac{1}{\sigma}+1\right)}$, and the fact that $m_{\varepsilon'} = 0$ for all $\varepsilon' \neq \bar{\varepsilon}$, this is:

$$\frac{m_{\bar{\varepsilon}}}{1-m_{\bar{\varepsilon}}} = \beta\left(1+r\right)^{2}\left(\sum_{\varepsilon'\neq\bar{\varepsilon}}\Pi_{\bar{\varepsilon}\varepsilon'}\left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\left(\frac{1}{\sigma}+1\right)} + \Pi_{\bar{\varepsilon}\bar{\varepsilon}}m_{\bar{\varepsilon}}\right)$$

Finally, using $\beta\left(1+r\right) = \frac{1}{\rho(\bar{\varepsilon})}$, we can also rewrite this expression as

$$\frac{m_{\bar{\varepsilon}}}{1-m_{\bar{\varepsilon}}} = \frac{1+r}{\rho\left(\bar{\varepsilon}\right)}\left(\sum_{\varepsilon'\neq\bar{\varepsilon}}\Pi_{\bar{\varepsilon}\varepsilon'}\left(\frac{\varepsilon'}{\bar{\varepsilon}}\right)^{-\left(\frac{1}{\sigma}+1\right)} + \Pi_{\bar{\varepsilon}\bar{\varepsilon}}m_{\bar{\varepsilon}}\right) \tag{A.94}$$

This is a quadratic equation that determines the MPC $m_{\bar{\varepsilon}}$ of the agent in state $\bar{\varepsilon}$ from primitives, from which we obtain $\lambda = \Pi_{\bar{\varepsilon}\bar{\varepsilon}}\left(1-m_{\bar{\varepsilon}}\right)\left(1+r\right)$.

**Implication for the M matrix of the ZL model.** Hence, $\mathbf{M}^{ZL} = \mathbf{I} - \left(\mathbf{I} - \left(1+r\right)\mathbf{L}\right)\mathbf{A}^{ZL}$ is a function of the same four parameters as $\mathbf{M}^{TABU}$: $\beta$, $r$, effective rate of decay of assets $\lambda = \Pi_{\overline{e}\overline{e}}\left(1-m_{\overline{e}}\right)\left(1+r\right)$, and effective hand-to-mouth share $\mu = 1 - \frac{\pi_{\overline{e}}}{\Pi_{\overline{e}\overline{e}}}\frac{\left(\overline{e}\right)^{1-\theta}}{\mathbb{E}[e^{1-\theta}]}$. $\mathbf{M}^{ZL}$ is almost—but not exactly—identical to $\mathbf{M}^{TABU}$; in particular:

$$M_{t0}^{ZL} = M_{t0}^{TABU} \quad t \geq 0; \quad M_{0s}^{ZL} = \zeta\cdot M_{0s}^{TABU} \quad s \geq 1$$

with $\zeta \equiv \frac{\frac{1}{\beta(1+r)} - \frac{\lambda}{1+r}}{1-\frac{\lambda}{1+r}} > 1$, since $\beta(1+r) < 1$. As the first equation shows, an unanticipated income shock is spent down in exactly the same way in the ZL and TABU models. While the HA-one model is not identical to ZL, this does suggest that the first columns, including further-out spending behavior, of the HA-one and TABU models are inherently related.

The key difference between $\mathbf{M}^{ZL}$ and $\mathbf{M}^{TABU}$ comes from the spending response to anticipated income shocks, where the ZL model implies a stronger spending response. This can be explained as follows. In both ZL and TABU, only the savers today respond to anticipated income tomorrow. But in ZL, unlike in TABU, some of this anticipated income will be received after switching to a different income state, with higher expected marginal utility. This has a larger effect on the savers' Euler equation and therefore their consumption today. This feature of ZL is also present in HA-one, and can be seen in figure 3(d) for anticipated shocks.

**Quasi-Toeplitz structure of $\mathbf{M}^{ZL}$ and $\mathbf{A}^{ZL}$.** Since $\mathbf{A}$ and $\mathbf{M}$ have the same structure for the ZL model as they do for the TABU model, we can follow the same arguments as in section D.2 to show that $A^{ZL}_{t,s} \to a^{ZL}_{t-s}$ and $M^{ZL}_{t,s} \to m^{ZL}_{t-s}$ for $t,s \to \infty$, and obtain exact expressions for $a^{ZL}_k$ and $m^{ZL}_k$. In particular $\mathbf{A}^{ZL}$ is quasi-Toeplitz with symbol $a^{ZL}(z) = a^{ZL}_+(z)\, a^{ZL}_-(z)$, where from (A.93), we calculate the symbols

$$a^{ZL}_+(z) = \frac{1-\mu}{1-\lambda z} \tag{A.95}$$

$$a^{ZL}_-(z) = \frac{1}{1-\beta\lambda z^{-1}}\left(\frac{\lambda}{1+r} - \frac{\beta\lambda z^{-1}}{\beta(1+r)}\right) = \frac{1}{1-\beta\lambda z^{-1}}\frac{\lambda(1-z^{-1})}{1+r} \tag{A.96}$$

and decomposing $a^{ZL}(z)$ into partial fractions, we find

$$\begin{aligned} a^{ZL} &= (1-\mu)\frac{\lambda}{1+r}\frac{1}{1-\beta\lambda^2}\left(\frac{1-\lambda}{1-\lambda z} - \frac{1-\beta\lambda}{1-\beta\lambda z^{-1}}z^{-1}\right) \\ &= (1-\mu)\frac{\lambda}{1+r}\frac{1}{1-\beta\lambda^2}\left((1-\lambda)\sum_{k=0}^{\infty}\lambda^k z^k - \left(\frac{1}{\beta\lambda}-1\right)\sum_{k=1}^{\infty}(\beta\lambda)^k z^{-k}\right) \end{aligned}$$

Hence, just like $\mathbf{A}^{TABU}$, the asymptotic Toeplitz column for $\mathbf{A}^{ZL}$ has a simple double-exponential form with rate of decay $\lambda$ on the right and $\beta\lambda$ on the left:

$$a^{ZL}_k = (1-\mu)\frac{\lambda}{1+r}\frac{1}{1-\beta\lambda^2}\begin{cases} -\left(\frac{1}{\beta\lambda}-1\right)(\beta\lambda)^{-k} & k<0 \\ (1-\lambda)\lambda^k & k\geq 0 \end{cases} \tag{A.97}$$

Similarly, we derive $m^{ZL}_k$ by calculating the symbol

$$\begin{aligned} m^{ZL}(z) &= \mu + \left(1-(1-(1+r)z)\frac{a^{ZL}(z)}{1-\mu}\right) \\ &= \mu + (1-\mu)\frac{1+\lambda\left(\beta\lambda - 1 - \frac{1}{1+r}\right) + \lambda\left(\frac{1}{1+r}-\beta\right)z^{-1}}{(1-\lambda z)(1-\beta\lambda z^{-1})} \end{aligned}$$

again decomposing $m^{ZL}(z)$ into partial fractions, we find

$$\begin{aligned} m^{ZL}(z) &= \mu + \frac{1-\mu}{1-\beta\lambda^2}\left((1-\lambda)\left(1-\frac{\lambda}{1+r}\right)\frac{1}{1-\lambda z} + \lambda\left(\frac{1}{\beta\lambda}-1\right)\left(\frac{1}{1+r}-\beta\lambda\right)\frac{\beta\lambda z^{-1}}{1-\beta\lambda z^{-1}}\right) \\ &= \mu + \frac{1-\mu}{1-\beta\lambda^2}\left((1-\lambda)\left(1-\frac{\lambda}{1+r}\right)\sum_{k=0}^{\infty}\lambda^k z^k + \lambda\left(\frac{1}{\beta\lambda}-1\right)\left(\frac{1}{1+r}-\beta\lambda\right)\sum_{k=1}^{\infty}(\beta\lambda)^k z^{-k}\right) \end{aligned}$$

Hence, just like for $\mathbf{M}^{TABU}$, the asymptotic Toeplitz column for $\mathbf{M}^{ZL}$ has the simple double-exponential form:

$$m_k^{ZL} = \mu 1_{k=0} + \frac{1-\mu}{1-\beta\lambda^2}\begin{cases}\lambda\left(\frac{1}{\beta\lambda}-1\right)\left(\frac{1}{1+r}-\beta\lambda\right)(\beta\lambda)^{-k} & k<0 \\ (1-\lambda)\left(1-\frac{\lambda}{1+r}\right)\lambda^k & k\geq 0\end{cases} \tag{A.98}$$

While the constants are slightly different from $m_k^{TABU}$ in (A.83), reflecting the greater degree of anticipation of income in ZL, the rate of decay of consumption after income is received is still exactly $\lambda$, and the rate of anticipation of future income is still exactly $\beta\lambda$. The limit is again attained quickly in practice, as is visible in figures 3 and D.1.

### D.5 Two-account HA model

The two-account model has the following Bellman equation. Denote $\varepsilon \equiv \frac{e^{1-\theta}}{\mathbb{E}[e^{1-\theta}]}$ for simplicity; this has the same Markov chain as $e$. Let $V_t\left(adj, \varepsilon, a_-^{liq}, a_-^{illiq}\right)$ be the value function for an agent coming into the period with adjustment opportunity $adj \in \{0,1\}$, income shock realization $\varepsilon$, and amounts $\left(a_-^{liq}, a_-^{illiq}\right)$ in liquid and illiquid accounts. For an adjuster, $adj = 1$, we have:

$$\begin{aligned} V_t\left(1, \varepsilon, a_-^{liq}, a_-^{illiq}\right) &= \max_{\tilde{c}, a^{liq}, a^{illiq}} u(\tilde{c}) + \beta\mathbb{E}\left[V_{t+1}\left(adj', \varepsilon', a^{liq}, a^{illiq}\right)|\varepsilon\right] \\ & \tilde{c} + a^{liq} + a^{illiq} = \varepsilon Z_t + (1+r_{t-1})(1-\zeta)a_-^{liq} + (1+r_{t-1})a_-^{illiq} \\ & a^{liq} \geq 0, a^{illiq} \geq 0 \end{aligned} \tag{A.99}$$

with $adj'$ distributed i.i.d with $Pr(adj'=1) = \nu$. For a non-adjuster, $adj = 0$, we have:

$$\begin{aligned} V_t\left(0, \varepsilon, a_-^{liq}, a_-^{illiq}\right) &= \max_{\tilde{c}, a} u(\tilde{c}) + \beta\mathbb{E}\left[V_{t+1}\left(adj', \varepsilon', a^{liq}, (1+r_t)a_-^{illiq}\right)|\varepsilon\right] \\ & \tilde{c} + a^{liq} = \varepsilon Z_t + (1+r_{t-1})(1-\zeta)a_-^{liq} \\ & a^{liq} \geq 0 \end{aligned} \tag{A.100}$$

where again $adj'$ is i.i.d with $Pr(adj'=1) = \nu$. We solve this problem for policy functions $\tilde{c}_t\left(adj, \varepsilon, a_-^{liq}, a_-^{illiq}\right)$, $a_t^{liq}\left(adj, \varepsilon, a_-^{liq}, a_-^{illiq}\right)$, and $a_t^{illiq}\left(adj, \varepsilon, a_-^{liq}, a_-^{illiq}\right)$. The first-order and envelope conditions of this problem are:

$$
\begin{aligned}
u'(\tilde{c}_{it}) &\geq \beta \mathbb{E}\left[V_{a^{liq},i,t+1}\right] && && \text{(A.101)}\\
u'(\tilde{c}_{it}) &\geq \beta \mathbb{E}\left[V_{a^{illiq},i,t+1}\right] && \text{if } adj_{it}=1 && \text{(A.102)}\\
V_{a^{liq},i,t} &= (1+r_{t-1})(1-\zeta)\,u'(\tilde{c}_{i,t}) && && \text{(A.103)}\\
V_{a^{illiq},i,t} &= (1+r_{t-1})\,u'(\tilde{c}_{i,t}) && \text{if } adj_{it}=1 && \text{(A.104)}\\
V_{a^{illiq},i,t} &= \beta(1+r_t)\,\mathbb{E}\left[V_{a^{illiq},i,t+1}\right] && \text{if } adj_{it}=0 && \text{(A.105)}
\end{aligned}
$$

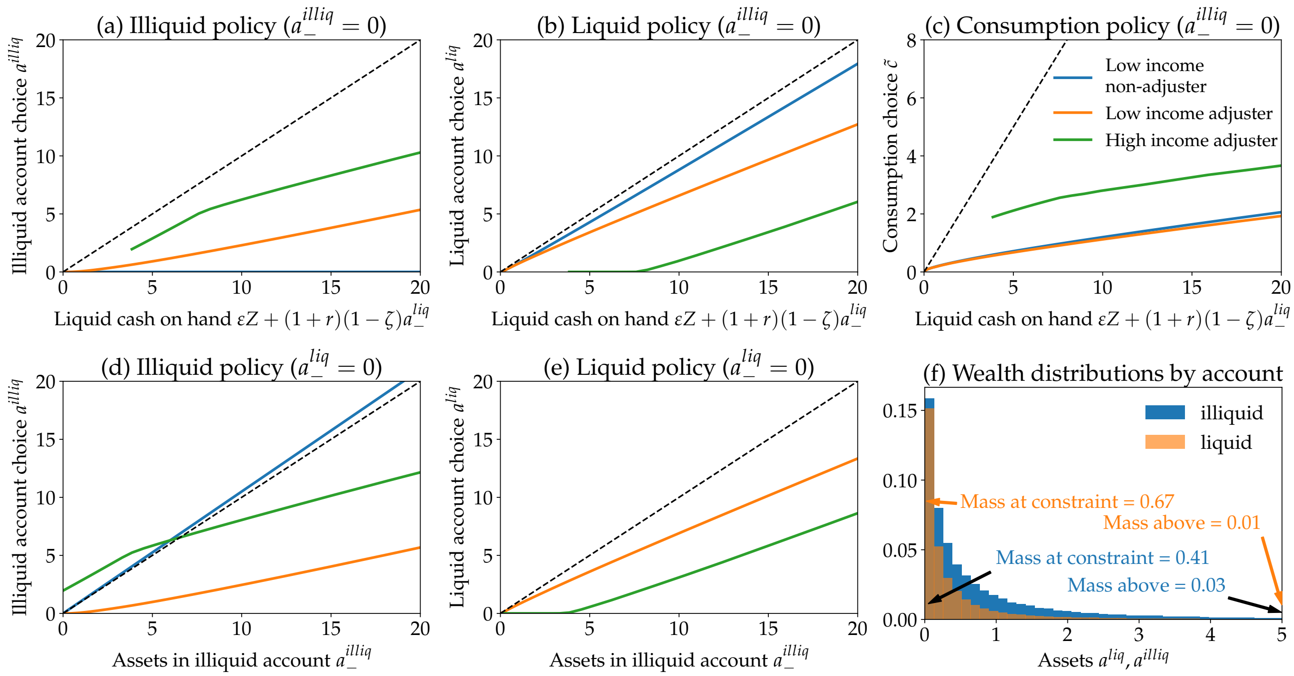
where, in the first, there is strict inequality when $a^{liq}=0$ and, in the second, there is strict inequality if $a^{illiq}=0$. Combining (A.101) and (A.103), we see that a consumer that is not at a borrowing constraint is always on an Euler equation for consumption between $t$ and $t+1$ with the liquid rate $(1+r_t)(1-\zeta)$. Additionally, combining (A.102) with (A.104) and (A.105), we see that a consumer with an opportunity to rebalance their assets is also on an Euler equation for consumption between $t$ and the next time he might adjust in the future at the higher rate $1+r_t$.

We solve the model as follows. We discretize $\varepsilon$ using an 11 point Markov chain, and $a^{liq}$ and $a^{illiq}$ using two doubly-exponentially spaced grids with 200 points each. We recursively solve for the marginal value functions $V_{a^{liq},t}\left(adj,\varepsilon,a_-^{liq},a_-^{illiq}\right)$ and $V_{a^{illiq},t}\left(adj,\varepsilon,a_-^{liq},a_-^{illiq}\right)$, as well as the policies $\tilde{c}_t\left(adj,\varepsilon,a_-^{liq},a_-^{illiq}\right)$, $a_t^{liq}\left(adj,\varepsilon,a_-^{liq},a_-^{illiq}\right)$, and $a_t^{illiq}\left(adj,\varepsilon,a_-^{liq},a_-^{illiq}\right)$ on this grid for $\left(adj,\varepsilon,a_-^{liq},a_-^{illiq}\right)$. Aggregating these policies by following the law of motion of the distribution, we obtain the the consumption and asset functions $\tilde{C}_t(\{Z_s\})$ and $A_t^{liq}(\{Z_s\})$ and $A_t^{illiq}(\{Z_s\})$, from which we form $C_t(\{Z_s\}) \equiv \tilde{C}_t(\{Z_s\}) + (1+r_t)\zeta A_{t-1}^{liq}(\{Z_s\})$ and $A_t(\{Z_s\}) \equiv A_t^{liq}(\{Z_s\}) + A_t^{illiq}(\{Z_s\})$. We build up the Jacobians of these functions, and therefore **M** and **A**, from from their "fake-news matrices", following the method in Auclert et al. (2021a).

Figure D.3 illustrates the steady-state policies and distributions for our calibrated HA-two model. The top panels report the policies for savings in the illiquid account $a^{illiq}$, liquid account $a^{liq}$, and consumption $\tilde{c}$ as a function of liquid cash-on-hand, $\varepsilon Z + (1+r)(1-\zeta)\,a_-^{liq}$, for an agent with no assets in their illiquid account, $a_-^{illiq}=0$, and different idiosyncratic states $\varepsilon$, for both adjusters $adj=1$ and non-adjusters $adj=0$. Low-income non-adjusters (in blue) have policies that are very similar to those in the high-liquidity one-account model HA-hi-liq in figure D.2: their consumption policy is concave, but they save quite aggressively—for them, the liquid account is the only option—with a policy that has a high slope. Low-income adjusters (in orange), have a very similar consumption policy, but start allocating around 40% of their savings to their illiquid account even at very low levels of cash on hand, as they are incentivized to do so by the high returns. By the same logic, higher-income agents allocate 100% of their savings to the illiquid account at low levels of cash on hand, leaving nothing in the liquid account and so allowing themselves to be "wealth hand-to-mouth" in the following periods. At higher levels of cash-on-hand, they are converting liquid assets into illiquid assets.

The bottom panels show the illiquid account choice policies for agents with no assets in their

Figure D.3: Steady-state policies and distributions in HA-two model



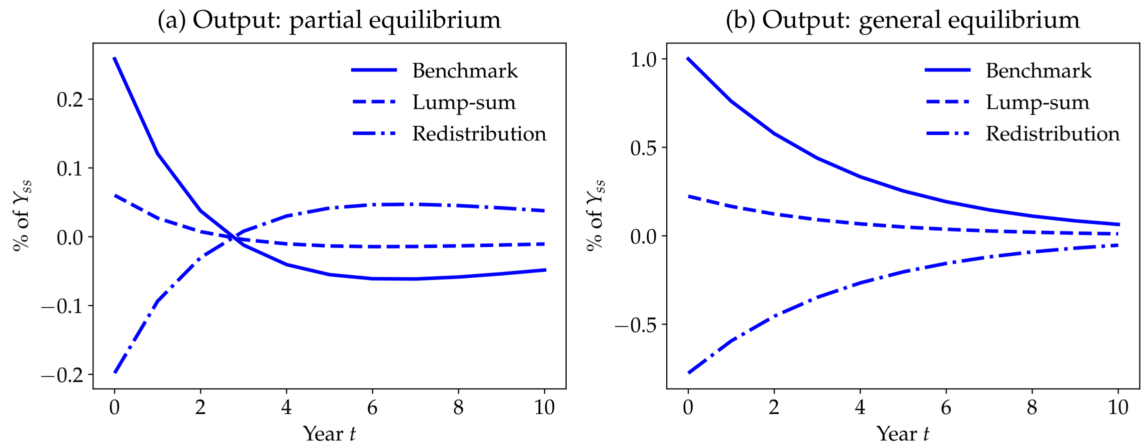

Note: Panels (a)–(d) report policies as a function of the level of liquid cash-on-hand $\varepsilon Z+(1+r)(1-\zeta)a_{-}^{liq}$ for an agent with no illiquid assets $a_{-}^{illiq}=0$. Panels (d)–(e) report policies as a function of the amount invested in the illiquid asset $a_{-}^{illiq}$ for an agent with no liquid assets $a_{-}^{liq}=0$. Panel (f) reports the stationary distributions of liquid and illiquid assets for $a\in(0,5)$, as well as the mass below and above these two end points. A low income consumer has $\epsilon=0.07$, and a high income consumer has $\epsilon=8.2$. Calibration parameters are in Table 2, and distributional statistics are in Table D.1.

liquid account, $a_{-}^{liq}=0$, as a function of the assets in their illiquid account $a_{-}^{illiq}$. Non-adjusters, of course, have to let their illiquid accounts grow at the illiquid account rate $1+r$. Since withdrawals only occur with probability $\nu=0.08$, this implies a natural tendency of illiquid account balances to grow. Adjusters rebalance their assets: at high levels of income they increase their illiquid account holdings if their illiquid balances are low, consistent with panels (a) & (b); and decrease them if their balances are high, consistent with maintaining a target level of illiquid assets. Here, agents with very high balances in their illiquid asset account convert about 30% of these balances to the liquid account.

Panel (e) show the resulting distribution of assets in the illiquid and the liquid account. Because of the high returns in the illiquid account and low probability of withdrawing once assets end up there, the distribution of assets in illiquid accounts is highly skewed to the right. Because about 30% of this is converted to liquid by adjusters, the distribution of liquid accounts is more concentrated towards 0 but still has some very large balances.

Table D.1 reports distributional statistics for this model. The share of agents that have no liquid assets is 57%, with 76% of agents having liquid assets below 0.1 (relative to average post-tax income of 0.61). 28% of agents, or about half of HtM agents, are "wealthy hand-to-mouth" (WHtM), with zero liquid assets but a positive amount of illiquid assets; this ratio is consistent with empirical estimates from the classification by Kaplan et al. (2014), but the levels are quite

Figure E.1: Comparing two ways to finance government spending: progressive vs. lump-sum taxation.




high. However, what matters to understand iMPCs is the distribution weighted by income. The table shows that 36% of income accrues to income that are HtM, and 22% to agents that are WHtM, these numbers are consistent with typical calibrations of two-account models (for instance Kaplan and Violante (2022), Table 5, column 2, report 39% of HtM agents and 26% of agents that are WHtM). This model has a very skewed wealth distribution, owing to the tendency of illiquid accounts to grow a lot before agents withdraw from them.

# E Appendix to section 5

## E.1 Proof of proposition 3

*Proof of proposition 3.* If $d\mathbf{G} = d\mathbf{T}$, then $d\mathbf{Y} = d\mathbf{G}$ solves equation (13), since with that guess we do have

$$d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} = d\mathbf{G} = d\mathbf{Y}$$

□

**Lump-sum taxation.** To illustrate the importance of the assumption of equal incidence of taxes and income for this result, we now solve two versions of the HA-one model: one with our benchmark tax rule, and one where all taxes are financed entirely lump-sum at the margin.[A-26]

Figure E.1 illustrates the effect of using lump-sum taxes. The solid line shows the benchmark effect on output of our benchmark AR(1) government spending shock, both in partial equilibrium on the left panel (where the effect on output is that of government spending net of the offsetting effect from the contemporaneous tax, ie $(\mathbf{I} - \mathbf{M})\, d\mathbf{G}$), and in general equilibrium on the right panel (where, after incomes have increased as a result of the increase in partial equilibrium demand, the

[A-26]This change in financing is only at the margin. To make sure that steady states are comparable, we retain our benchmark progressive fiscal rule otherwise.

remaining effect is just $d\mathbf{G}$, consistent with proposition 3). The dashed line shows the effect when lump-sum taxes are used to finance government spending instead. This dramatically lowers the impact on output, both in partial equilibrium (from 0.23 to 0.04), and in general equilibrium (from 1 to 0.14).

We can use the generalized IKC to understand this result. Conceptually, the lump-sum tax experiment is equivalent to combining our benchmark government spending shock with an additional redistribution shock from low-productivity to high-productivity households in the periods of taxation. To see this, note that, manipulating the generalized IKC equation (19) and using proposition 3, we have:

$$d\mathbf{Y} = \mathcal{M}\left(d\mathbf{G} - \mathbf{M}^T d\mathbf{G}\right) = \mathcal{M}\left(d\mathbf{G} - \mathbf{M}d\mathbf{G}\right) + \mathcal{M}\left(\mathbf{M} - \mathbf{M}^T\right)d\mathbf{G} = d\mathbf{G} + \mathcal{M}\left(\mathbf{M} - \mathbf{M}^T\right)d\mathbf{G}$$

Hence, the general equilibrium output effect is the sum of our benchmark effect $d\mathbf{G}$, and the general equilibrium effect of a partial equilibrium redistribution shock that levies a lump-sum tax of $dG_t$ in each period and uses the proceeds to lower proportional taxes in the same period. The effect of that redistribution shock alone is plotted in the dash-dot line. Because the iMPCs for lump-sum transfers are substantially above those for progressive transfers in our calibration (the static MPC is 0.72 for a lump-sum transfer vs 0.51 for a progressive transfer), a difference in tax incidence translates into substantially different partial equilibrium and therefore general equilibrium multipliers.[A-27]

## E.2 Proof of proposition 4

*Proof of proposition 4.* Rewriting equation (13) as

$$d\mathbf{Y} - d\mathbf{G} = \mathbf{M}\left(d\mathbf{G} - d\mathbf{T}\right) + \mathbf{M}\left(d\mathbf{Y} - d\mathbf{G}\right)$$

and applying proposition 1 implies that the solution for $d\mathbf{Y} - d\mathbf{G}$ must be given by

$$d\mathbf{Y} - d\mathbf{G} = \mathcal{M}\mathbf{M}\left(d\mathbf{G} - d\mathbf{T}\right)$$

This delivers equation (30). Since $d\mathbf{Y} - d\mathbf{G} = d\mathbf{C}$, we have that $d\mathbf{C} = \mathcal{M}\mathbf{M}\left(d\mathbf{G} - d\mathbf{T}\right)$. □

## E.3 Determinacy and multipliers for analytical models

*Proof of proposition 5.* In the RA model, the $\mathbf{A}$ matrix is given by (A.60), which is $\mathbf{A}^{RA} = \mathbf{K}\left(\mathbf{I} - \mathbf{M}^{RA}\right) = \mathbf{K}\left(\mathbf{I} - \frac{\mathbf{1}\mathbf{q}'}{\mathbf{q}'\mathbf{1}}\right)$. Note that $\mathbf{A}\cdot\mathbf{1} = \mathbf{K}\left(\mathbf{1} - \mathbf{1}\right) = 0$, so $\mathbf{A}$ is not injective since the vector $\mathbf{1}$ is in its kernel.

For any $(d\mathbf{G}, d\mathbf{T})$ satisfying $\mathbf{q}'d\mathbf{G} = \mathbf{q}'d\mathbf{T}$, we have $\mathbf{M}^{RA}\left(d\mathbf{G} - d\mathbf{T}\right) = \frac{1}{\mathbf{q}'\mathbf{1}}\mathbf{1}\mathbf{q}'\left(d\mathbf{G} - d\mathbf{T}\right) = 0$, so the IKC (13) rewrites

$$d\mathbf{Y} - d\mathbf{G} = \mathbf{M}^{RA}\left(d\mathbf{Y} - d\mathbf{G}\right)$$

[A-27]Note that the present value of all partial equilibrium impulses is zero, consistent with the general result proved in section A.8.

Hence, the entire set of solutions to (13) is given by $d\mathbf{Y} = d\mathbf{G} + \text{Ker}\left(\mathbf{I} - \mathbf{M}^{RA}\right)$. Consider any element $\mathbf{v} \in \text{Ker}\left(\mathbf{I} - \mathbf{M}^{RA}\right)$: it must satisfy $\mathbf{v} = \frac{\mathbf{q}'\mathbf{v}}{\mathbf{q}'\mathbf{1}}\mathbf{1}$. Hence, $\mathbf{v}$ is proportional to the vector $\mathbf{1}$. It follows that $\text{Ker}\left(\mathbf{I} - \mathbf{M}^{RA}\right)$ has dimension 1 and that all solutions to the IKC (13) are

$$d\mathbf{Y} = d\mathbf{G} + \lambda \mathbf{1}, \quad \lambda \in \mathbb{R}$$

Now, recall that we are restricted to consider shocks satisfying $\lim_{t\to\infty} dG_t = 0$. Then, the solution with $\lambda = 0$ is the unique solution satisfying $\lim_{t\to\infty} dY_t = 0$, which is the solution with $\lambda = 0$. This proves that $d\mathbf{Y} = d\mathbf{G}$ is the unique solution with this equilibrium selection imposed. □

*Proof of proposition 6.* In the TA model, the $\mathbf{A}$ matrix is given by $\mathbf{A}^{TA} = (1-\mu)\mathbf{A}^{RA} + \mu\mathbf{I}$, so $\mathbf{A}^{TA}\mathbf{1} = 0$ and $\mathbf{A}^{TA}$ is not injective.

For any $(d\mathbf{G}, d\mathbf{T})$ satisfying $\mathbf{q}'d\mathbf{G} = \mathbf{q}'d\mathbf{T}$, using $\mathbf{M}^{TA} = (1-\mu)\mathbf{M}^{RA} + \mu\mathbf{I}$, so $\mathbf{I} - \mathbf{M}^{TA} = (1-\mu)\left(\mathbf{M}^{RA} - \mathbf{I}\right)$, we can rewrite the IKC $\left(\mathbf{I} - \mathbf{M}^{TA}\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T}$ as:

$$\left(\mathbf{I} - \mathbf{M}^{RA}\right) d\mathbf{Y} = \frac{1}{1-\mu}\left(d\mathbf{G} - \mu d\mathbf{T}\right) - \mathbf{M}^{RA} d\mathbf{T}$$

Hence, this equation has the same set of solutions as the IKC for the RA model, with $d\mathbf{G}$ replaced by $\frac{1}{1-\mu}\left(d\mathbf{G} - \mu d\mathbf{T}\right)$. It follows that the set of solutions is

$$d\mathbf{Y} = \frac{1}{1-\mu}\left(d\mathbf{G} - \mu d\mathbf{T}\right) + \lambda \mathbf{1}, \quad \lambda \in \mathbb{R}$$

and that, for shocks satisfying $\lim_{t\to\infty} dG_t = \lim_{t\to\infty} dT_t = 0$, the unique solution satisfying $\lim_{t\to\infty} dY_t = 0$ features $\lambda = 0$. □

*Proof of proposition 7.* By proposition 2, the set of solutions to the IKC (13) is the set of solutions to $\mathbf{A}\left(d\mathbf{Y} - d\mathbf{T}\right) = d\mathbf{B}$.

As shown in appendix D.2, $\mathbf{A}^{TABU}$ can be written as the product of a lower and an upper triangular Toeplitz operators, $\mathbf{A}^{TABU} = \mathbf{T}(a_+)\mathbf{T}(a_-)$. If both $\mathbf{T}(a_+)$ and $\mathbf{T}(a_-)$ are invertible, then $\mathbf{A}^{TABU}$ is invertible and equal to $\mathbf{T}\left(a_-^{-1}\right)\mathbf{T}\left(a_+^{-1}\right)$. Since this the product of an upper and a lower triangular Toeplitz matrix, by standard results (eg Böttcher and Grudsky 2005), $\left(\mathbf{A}^{TABU}\right)^{-1}$ is therefore itself Toeplitz and equal to $\mathbf{T}\left(a_-^{-1}a_+^{-1}\right)$.[A-28] Furthermore, the symbols of $\mathbf{T}(a_+)$ and $\mathbf{T}(a_-)$ are given by equations (A.80) and (A.81).

We know that $\mathbf{T}(a_+)$ and $\mathbf{T}(a_-)$ are invertible if and only if their symbols have no zero on the unit circle and their winding number is zero; where the winding number is the difference between zeros and poles inside the unit circle. Given (A.80), $a_+(z)$ has a single pole $\frac{1}{\lambda}$ outside the unit circle and no zero, it is always invertible. Next, given (A.81), $a_-(z)$ has one pole $\beta\lambda$ inside the unit circle, and its zero is $\beta(1+r)$, which is strictly inside the unit circle provided $\beta(1+r) < 1$. Combining,

[A-28] On the other hand, as discussed in appendix D.2, $\mathbf{A}^{TABU}$ is not Toeplitz. Instead, it is *quasi-Toeplitz*, ie equal to the sum of a Toeplitz matrix with symbol $a_+a_-$ and a compact matrix.

we find that $\mathbf{A}^{TABU}$ is invertible if and only if $\beta(1+r) < 1$. In that case, its inverse is the Toeplitz matrix with symbol $a_-^{-1}(z)\,a_+^{-1}(z)$, where

$$a_-^{-1}(z)\,a_+^{-1}(z) = \frac{1}{1-\mu}\frac{1+r}{\lambda}\frac{(1-\lambda z)\left(1-\beta\lambda z^{-1}\right)}{1-\beta(1+r)z^{-1}}$$

Since

$$\frac{\left(\frac{1+r}{\lambda}\right)(1-\lambda z)\left(1-\beta\lambda z^{-1}\right)}{1-\beta(1+r)z^{-1}} - (1-(1+r)\,z) \quad = \quad (1+r)\,\frac{\left(1-\frac{\lambda}{1+r}\right)\left(\frac{1}{\lambda}-\beta\,(1+r)\right)}{1-\beta(1+r)z^{-1}}$$

we have that

$$\begin{aligned} a_-^{-1}(z)\,a_+^{-1}(z) &= \frac{1}{1-\mu}\frac{(1+r)(1-\frac{\lambda}{1+r})(\frac{1}{\lambda}-\beta(1+r))}{1-\beta(1+r)z^{-1}} + \frac{1}{1-\mu}(1-(1+r)\,z) \\ &= \frac{1+r}{1-\mu}(1-\frac{\lambda}{1+r})(\frac{1}{\lambda}-\beta(1+r))\sum_{k\geq 0}\beta^k(1+r)^k z^{-k} + \frac{1-(1+r)\,z}{1-\mu} \end{aligned}$$

so that we have the explicit expression

$$\left(\mathbf{A}^{TABU}\right)^{-1} = \frac{1+r}{1-\mu}(1-\frac{\lambda}{1+r})(\frac{1}{\lambda}-\beta(1+r))\sum_{k\geq 0}\beta^k(1+r)^k\mathbf{F}^k + \frac{(\mathbf{I}-(1+r)\,\mathbf{L})}{1-\mu}$$

The solution to the IKC is $d\mathbf{Y} = \left(\mathbf{A}^{TABU}\right)^{-1}d\mathbf{B} + d\mathbf{T}$, which gives:

$$d\mathbf{Y} = \frac{(1-(1+r)\,\mathbf{L})}{1-\mu}d\mathbf{B} + d\mathbf{T} + \frac{1+r}{1-\mu}(1-\frac{\lambda}{1+r})(\frac{1}{\lambda}-\beta(1+r))\sum_{k\geq 0}\beta^k(1+r)^k\mathbf{F}^k d\mathbf{B}$$

however, since $(1-(1+r)\,\mathbf{L})\,d\mathbf{B} + d\mathbf{T} = d\mathbf{G}$, we have that

$$\frac{(1-(1+r)\,\mathbf{L})}{1-\mu}d\mathbf{B} + d\mathbf{T} = \frac{d\mathbf{G}-d\mathbf{T}}{1-\mu} + d\mathbf{T} = d\mathbf{G} + \frac{\mu}{1-\mu}(d\mathbf{G}-d\mathbf{T})$$

so we also have

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}(d\mathbf{G}-d\mathbf{T}) + \frac{1+r}{1-\mu}(1-\frac{\lambda}{1+r})\left(\frac{1}{\lambda}-\beta(1+r)\right)\sum_{k\geq 0}\beta^k(1+r)^k\mathbf{F}^k d\mathbf{B} \tag{A.106}$$

which is the expression in the text. In the TA case with $\lambda = 1$ and $\beta = \frac{1}{1+r}$, we have $\frac{1}{\lambda} - \beta(1+r) = 0$ and recover proposition 6. □

*Proof of corollary 1.* Fix $\beta$, $r$, and a calibration for $M_{00} = \mu + (1-\mu)\left(1-\frac{\lambda}{1+r}\right)$. Recall that $M_{10} = (1-\mu)\left(1-\frac{\lambda}{1+r}\right)\lambda$. This implies in particular that

$$M_{10} = (1+r)\,(1-M_{00})\left(1-\frac{\lambda}{1+r}\right)$$

which says that the MPC of the BU agent $1-\frac{\lambda}{1+r}$ can be recovered by taking the ratio of the amount spent in period 1, $M_{10}$, to the incoming assets in period 1, $(1+r)(1-M_{00})$. Given this, we can solve for $\lambda$ and $\mu$ as follows:

$$\lambda = 1+r-\frac{M_{10}}{1-M_{00}} \tag{A.107}$$

$$\mu = M_{00}-\frac{M_{10}}{\lambda} \tag{A.108}$$

Hence, conditional on $r$ and $M_{00}<1$, raising $M_{10}$ lowers $\lambda$ and lowers $\mu$.

Now, since $\mathbf{q}'(d\mathbf{G}-d\mathbf{T})=0$, the cumulative multiplier implied by (A.106) is

$$\frac{\mathbf{q}'d\mathbf{Y}}{\mathbf{q}'d\mathbf{G}} = 1+(1+r)\left(\frac{1-\frac{\lambda}{1+r}}{1-\mu}\right)\left(\frac{1}{\lambda}-\beta(1+r)\right)\frac{\sum_{k\geq 0}\beta^k(1+r)^k\mathbf{q}'\mathbf{F}^k d\mathbf{B}}{\mathbf{q}'d\mathbf{G}}$$

When $d\mathbf{B}\geq 0$ and $dB_t>0$ for some $t$, this is a strictly increasing function of

$$f\equiv\frac{1}{1-\mu}(1-\frac{\lambda}{1+r})(\frac{1}{\lambda}-\beta(1+r))$$

Given (A.107) and (A.108), we have:

$$1-\mu=1-M_{00}+\frac{M_{10}}{\lambda}=(1-M_{00})\left(1+\frac{1+r}{\lambda}-1\right)=(1-M_{00})\frac{1+r}{\lambda} \tag{A.109}$$

so that

$$\begin{aligned} f(\lambda) &= \frac{1}{1-M_{00}}\frac{\lambda}{1+r}(1-\frac{\lambda}{1+r})(\frac{1}{\lambda}-\beta(1+r)) \\ &= \frac{1}{1-M_{00}}\left(1-\frac{\lambda}{1+r}\right)\left(\frac{1}{1+r}-\beta\lambda\right) \\ &= \frac{\beta/(1+r)}{1-M_{00}}(\lambda-(1+r))\left(\lambda-\frac{1}{\beta(1+r)}\right) \end{aligned}$$

Hence $f(\lambda)$ is a convex quadratic in $\lambda$. Provided that $r>0$ (which is assumed throughout) and $\beta(1+r)\leq 1$ (which is required for determinacy), the two roots of $f$ are $1+r>1$ and $\frac{1}{\beta(1+r)}\geq 1$, so $f$ is decreasing for $\lambda\in[0,1]$.

To conclude, conditional on $r>0$ (with $\beta(1+r)<1$) and $M_{00}<1$, raising $M_{10}$ lowers $\lambda$ and raises $f$, so raises the cumulative multiplier for any $d\mathbf{B}\geq 0$ and $dB_t>0$ for some $t$, as we set out to prove. □

In addition, we state and prove the following equivalent of proposition 7 for the ZL model.

**Proposition 11** (Fiscal policy in the ZL model)**.** *Consider a ZL model with parameters $\lambda$, $\mu$, $\beta$ and $r$. Then, for given fiscal policy $\{dG_t, dT_t\}$ generating a path of debt $dB_t=\sum_{s\leq t}(1+r)^s(dG_s-dT_s)$, there exists a unique solution $\{dY_t\}$ to the IKC (13) with the property that $\lim_{t\to 0}dY_t=0$ whenever*

$\lim_{t\to 0} dG_t = \lim_{t\to 0} dT_t = 0$, *and it is given by*

$$dY_t = dG_t + \frac{\mu}{1-\mu}\left(dG_t - dT_t\right) - \frac{1-\beta\left(1+r\right)}{1-\mu} dB_t + \left(1+r\right)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda}-1\right)\sum_{s=0}^{\infty} dB_{t+s} \quad \text{(A.110)}$$

Note that the formula for output in the ZL model is very similar to that in (31), but future debt is undiscounted rather than discounted at rate $\beta(1+r)$. This results from the slightly larger anticipatory effects in the ZL model, as discussed in section D.4.

*Proof of proposition 11.* We proceed analogously to the proof for the TABU model. Since $\mathbf{A}^{ZL}$ is the product of a lower and an upper diagonal Toeplitz matrix $\mathbf{T}\left(a_+\right)\mathbf{T}\left(a_-\right)$, its inverse $\left(\mathbf{A}^{ZL}\right)^{-1}$ is exactly Toeplitz. From (A.93), we calculate the symbols

$$\begin{aligned} a_+\left(z\right) &= \frac{1-\mu}{1-\lambda z} \\ a_-\left(z\right) &= \frac{1}{1-\beta\lambda z^{-1}}\left(\frac{\lambda}{1+r} - \frac{\beta\lambda z^{-1}}{\beta(1+r)}\right) = \frac{1}{1-\beta\lambda z^{-1}}\frac{\lambda(1-z^{-1})}{1+r} \end{aligned}$$

Since $a_+\left(z\right)$ has a single pole $\frac{1}{\lambda}$ outside the unit circle and no zero, $\mathbf{T}\left(a_+\right)$ is invertible. Since $a_+\left(z\right)$ has a pole $\beta\lambda$ inside the unit circle and a zero (equal to 1) on the unit circle, it is on the border of invertibility. The inverse limit has Toeplitz symbol:

$$a_-^{-1}\left(z\right)a_+^{-1}\left(z\right) = \frac{1}{1-\mu}(1-\lambda z)(1-\beta\lambda z^{-1})\frac{1+r}{\lambda}\frac{1}{1-z^{-1}}$$

Since

$$\begin{aligned} & (1-\lambda z)(1-\beta\lambda z^{-1})\frac{1+r}{\lambda}\frac{1}{1-z^{-1}} - \left(1-\left(1+r\right)z\right) \\ =\ & \frac{\frac{1+r}{\lambda}(1-\beta\lambda z^{-1} - \lambda z + \beta\lambda^2) - \left(1-\left(1+r\right)z - z^{-1} + 1 + r\right)}{1-z^{-1}} \\ =\ & \frac{\frac{1+r}{\lambda}(1+\beta\lambda^2) - \left(1+r\right)\left(1+\beta\right)}{1-z^{-1}} - \left(1-\beta\left(1+r\right)\right) \\ =\ & \frac{\left(1+r\right)\left(\frac{1}{\lambda}-1\right)\left(1-\beta\lambda\right)}{1-z^{-1}} - \left(1-\beta\left(1+r\right)\right) \end{aligned}$$

we have that

$$a_-^{-1}\left(z\right)a_+^{-1}\left(z\right) = \left(1+r\right)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda}-1\right)\sum_{k\geq 0} z^{-k} - \frac{1-\beta\left(1+r\right)}{1-\mu} + \frac{\left(1-\left(1+r\right)z\right)}{1-\mu}$$

Hence, the inverse limit is

$$\left(\mathbf{A}^{ZL}\right)^{-1} = \left(1+r\right)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda}-1\right)\sum_{k\geq 0}\mathbf{F}^k - \frac{1-\beta\left(1+r\right)}{1-\mu}\mathbf{I} + \frac{\left(\mathbf{I}-\left(1+r\right)\mathbf{L}\right)}{1-\mu}$$

The solution to the IKC with the property that $\lim_{t\to 0} dY_t = 0$ whenever $\lim_{t\to 0} dG_t = \lim_{t\to 0} dT_t = 0$ is $d\mathbf{Y} = d\mathbf{T} + \left(\mathbf{A}^{ZL}\right)^{-1} d\mathbf{B}$. Using the fact that $d\mathbf{T} + \frac{(1-(1+r)\mathbf{L})}{1-\mu} d\mathbf{B} = d\mathbf{G} + \frac{\mu}{1-\mu}\left(d\mathbf{G} - d\mathbf{T}\right)$, we also have:

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu}\left(d\mathbf{G} - d\mathbf{T}\right) - \frac{1-\beta\left(1+r\right)}{1-\mu} d\mathbf{B} + \left(1+r\right)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda} - 1\right)\sum_{s=0}^{\infty} \mathbf{F}^k d\mathbf{B}$$

which is the expression in (A.110). □

We also have the following equivalent of corollary 1 for this model:

**Corollary 3.** *Holding $\beta$, $r$, and $M_{00}$ fixed in the ZL model, with $\beta\left(1+r\right) \leq 1$, a higher $M_{10}$ increases the cumulative multiplier whenever $d\mathbf{B} \geq 0$ and $dB_t > 0$ for some $t$.*

*Proof.* This follows the same steps as the proof of corollary 1. The relationship between $\left(M_{00}, M_{10}\right)$ and $\left(\mu, \lambda\right)$ is the same as in the TABU model, and given by (A.107)–(A.108), and so we still have (A.109). The cumulative multiplier implied by (A.110) is

$$\frac{\mathbf{q}' d\mathbf{Y}}{\mathbf{q}' d\mathbf{G}} = 1 - \frac{1-\beta\left(1+r\right)}{1-\mu}\frac{\mathbf{q}' d\mathbf{B}}{\mathbf{q}' d\mathbf{G}} + \left(1+r\right)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda} - 1\right)\frac{\sum_{k\geq 0} \mathbf{q}' \mathbf{F}^k d\mathbf{B}}{\mathbf{q}' d\mathbf{G}}$$

When $d\mathbf{B} \geq 0$ and $dB_t > 0$ for some $t$, the first term is

$$-\frac{1-\beta\left(1+r\right)}{1-\mu} = -\frac{1-\beta\left(1+r\right)}{1-M_{00}}\frac{\lambda}{1+r}$$

which is strictly decreasing in $\lambda$ when $\beta\left(1+r\right) \leq 1$, and the second term is a strictly increasing function of

$$f\left(\lambda\right) \equiv \left(1+r\right)\frac{1-\beta\lambda}{1-\mu}\left(\frac{1}{\lambda} - 1\right) = \frac{1}{1-M_{00}}\left(1-\beta\lambda\right)\left(1-\lambda\right)$$

which is a convex quadratic in $\lambda$ with roots at 1 and $\frac{1}{\beta} \geq 1+r > 1$, and so is decreasing for $\lambda \in [0,1]$. To conclude, conditional on $r > 0$, $\beta\left(1+r\right) \leq 1$, and $M_{00} < 1$, raising $M_{10}$ lowers $\lambda$ and raises the cumulative multiplier for any $d\mathbf{B} \geq 0$ and $dB_t > 0$ for some $t$, as we set out to prove. □

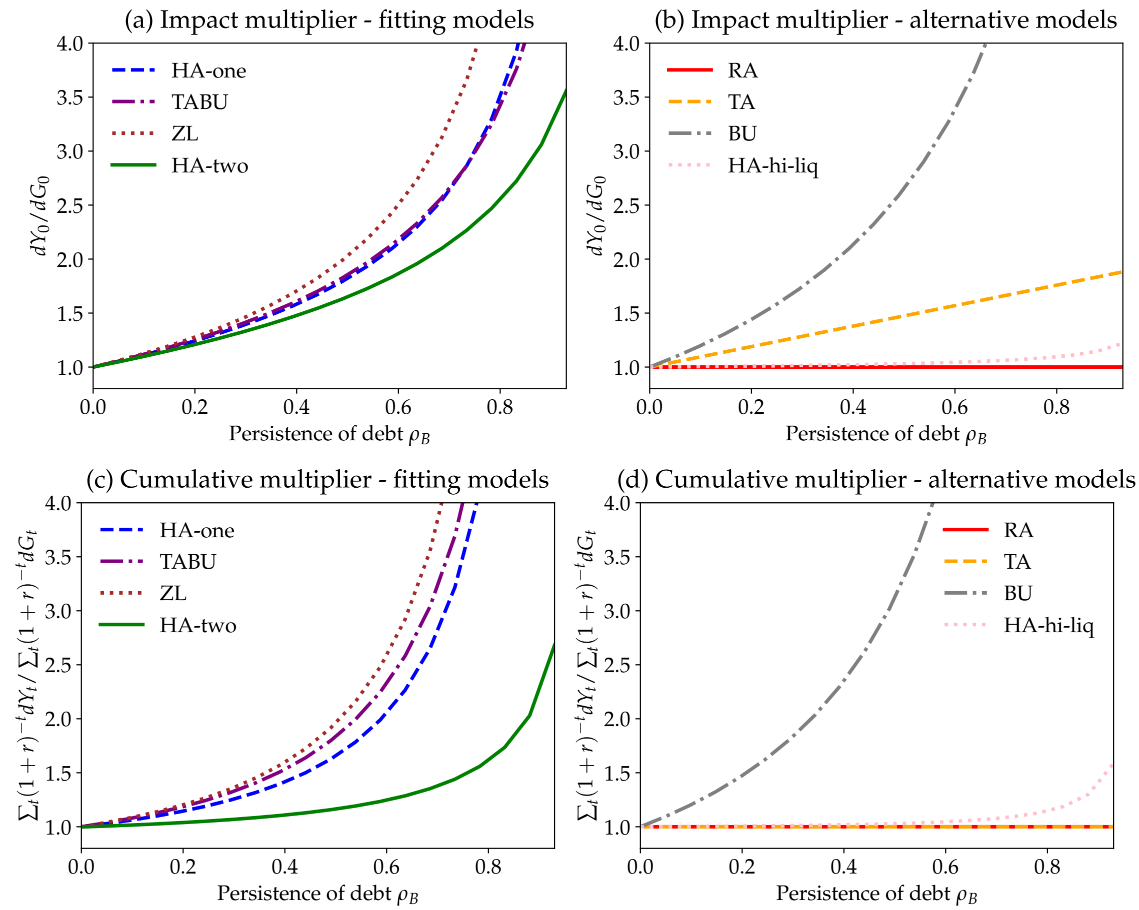
### E.4 Numerical multipliers across all models

In figure E.2 we show the impact and cumulative multipliers of all eight IKC models introduced in section 4. We see that ZL is very similar to TABU; BU is more extreme than TABU since it has an even greater iMPC $M_{10}$; and HA-hi-liq is very close to the RA model due to approximate aggregation.

### E.5 Nonlinearities and state dependence in fiscal multipliers

In this section, we examine nonlinearities and state dependence in the effects of fiscal policy in our benchmark models.

Figure E.2: Multipliers across all eight models in the IKC environment

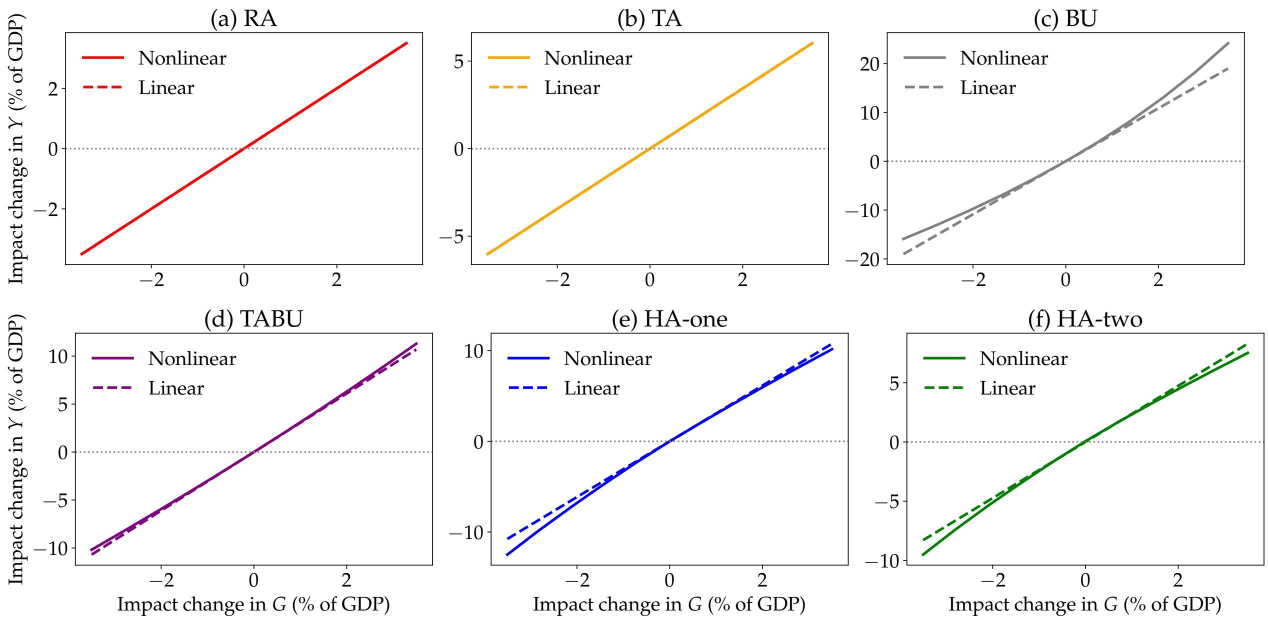



To investigate nonlinearities, we solve for the perfect-foresight transition in response to shocks to fiscal policy of different sizes. Our benchmark fiscal policy shock assumes a path $G_t - G_{ss} = \sigma \cdot \rho^t$, and $B_t - B_{ss} = \sigma\left(\rho_B\left(B_t - B_{ss}\right) + G_t - G_{ss}\right)$. The limit $\sigma \to 0$ is the linear impulse response that we solve for in the main text. We do this in different models and for different values of $\sigma$.

Figure E.3 summarizes the results by reporting the impact effect on output $Y_0 - Y_{ss}$ against the initial effect on government spending relative to GDP $G_0 - G_{ss} = \epsilon$. Since the results in proposition 5 and 6 hold nonlinearly, RA and TA do not have any nonlinearity.

Figure E.3 shows the impact effect on output $Y_0 - Y_{ss}$ for an unanticipated MIT shock to government spending $G_0 - G_{ss}$ of different sizes relative to GDP, using the same calibration $\rho_G = \rho_B = 0.76$ for persistence as in figure 6. In BU and TABU, when calibrated with quadratic utility for holding bonds $\chi$, the effects of government spending are convex in size: larger positive shocks have larger multipliers, and large negative shocks have smaller multipliers. The dashed line is the linear approximation, whose slope is the model's fiscal multiplier reported in the third row of table 4. In HA-one and HA-two, the opposite is true: the effects of government spending are concave in size. Observe, however, that the nonlinearities are slight in all cases: for instance, in the HA-two model, a decline of government spending of 4% of GDP has an impact of 19% on output,

Figure E.3: Nonlinearity in the effect of government spending in our models



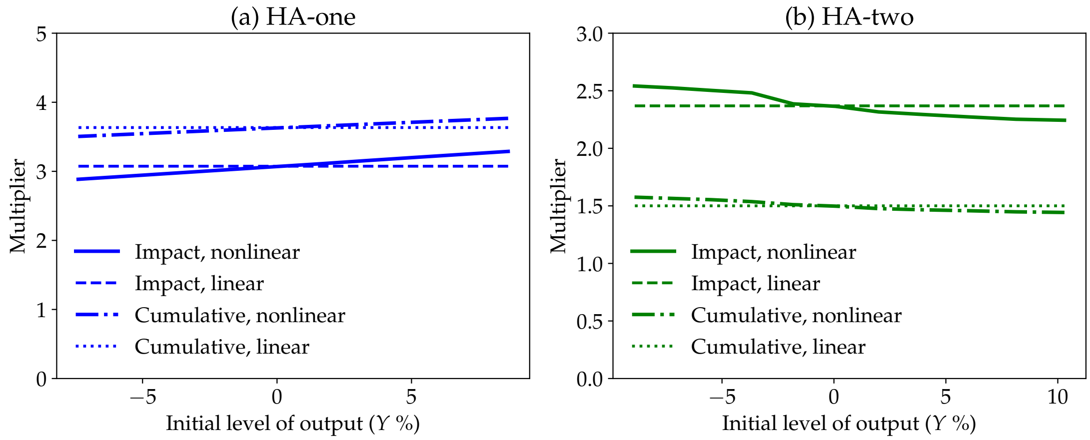

relative to the value of 14% predicted by the multiplier of 3.5 (see table 4), a difference of 25%.

Next, we investigate the extent of state dependence in impulse responses. Since this is most natural to do in models in which the state is a full distribution, we restrict our attention to the HA-one and HA-two models.

We consider the following experiment. In year $t = -1$, a negative demand shock of size $\epsilon$ hits the economy and a recession begins. In year $t = 0$, government spending unexpectedly increases by a small amount. We evaluate the impact and cumulative multiplier on that government spending as a function of the size of the recession in period 1. The demand shock is a shock to household's discount factors $\beta$. Formally, we solve a perfect-foresight transition to the first shock at $t = -1$ and obtain an impulse response for output as well as a distribution over state variables $D_0\left(\epsilon, a_{-}^{liq}, a_{-}^{illiq}\right)$. Then, starting from this distribution, we solve for a second perfect-foresight transition in which the initial shock continues, but there is also a shock to government spending and taxes of the usual shape, scaled so that its impact effect on government spending is 0.1% of output. We report multipliers as the effect on output relative to the baseline transition.

Figure E.4 displays impact and cumulative multipliers as a function of the size of the recession induced by the demand shock. In the HA-one model, there is a modest amount of state dependence, and fiscal multipliers are smaller in recessions than in booms. When initial output is 10% below normal, the impact multiplier is 10% smaller (6.9 rather than 7.6), and the cumulative multiplier is 3% smaller (16.4 rather than 16.9) than when the economy is at steady state. In the HA-two model, by contrast, there is a small amount of state dependence, and fiscal multipliers are larger in recessions than in booms. When initial output is 10% below normal, both initial and cumulative multipliers are 2% larger than when the economy is at steady state.

Figure E.4: State dependence in the effects of government spending in HA-one and HA-two




## E.6 Cognitive discounting

To implement cognitive discounting in our model, we follow the procedure discussed in Auclert, Rognlie and Straub (2020) for obtaining the solution to a model with informational rigidities, starting from the full-information solution. For Gabaix (2020)'s cognitive discounting model, that paper shows that, starting from the full-information Jacobian $\mathbf{J}$, cognitive discounting implies a new Jacobian $\mathbf{J}^\delta$ given by

$$J^\delta_{t,s} = \begin{cases} \delta^s J_{t,s} & t = 0, s \geq 0 \\ \delta^s \left(J_{t,s} - J_{t-1,s-1}\right) + J^\delta_{t-1,s-1} & t > 0, s > 0 \end{cases} \tag{A.111}$$

(see formula in Auclert et al. 2020, appendix D.3). This keeps the first column of the Jacobian $\mathbf{J}$ constant and discounts the first row at rate $\delta$. An alternative is to express this in terms of the fake news matrix, defined as $F_{t,s} = J_{t,s}$ for $t = 0$ or $s = 0$ and $F_{t,s} \equiv J_{t,s} - J_{t-1,s-1}$ for $t \geq 1, s \geq 1$ (see Auclert et al. 2021a). Then, we simply have

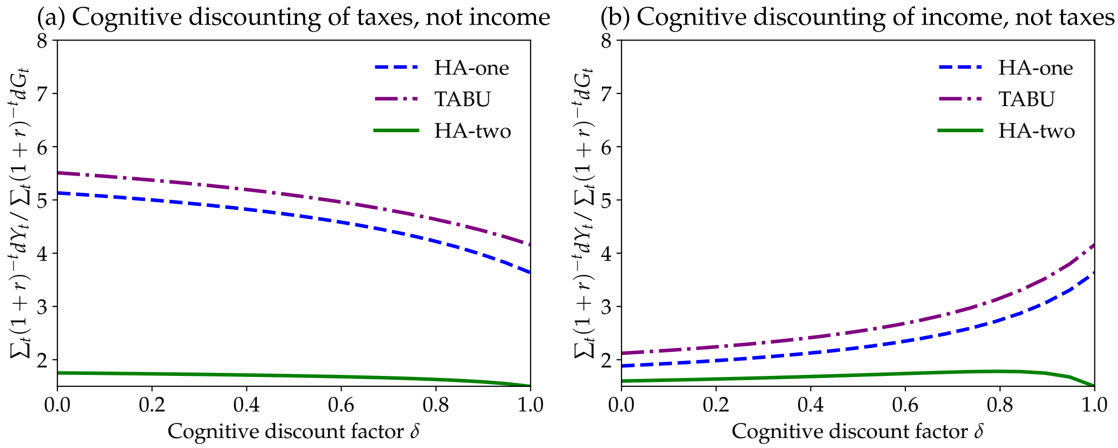
$$F^\delta_{t,s} = \delta^s F_{t,s} \qquad \forall t, s \tag{A.112}$$

and we can then build the Jacobian $\mathbf{J}^\delta$ from the recursion $J^\delta_{t,s} = \sum_{k=0}^{\min\{s,t\}} F^\delta_{t-k,s-k}$.[A-29]

We generate figure 6 by applying cognitive discounting to the $\mathbf{M}$ matrix of each of our models, delivering $\mathbf{M}^\delta$, and then solving for $\left(\mathbf{I} - \mathbf{M}^\delta\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^\delta d\mathbf{T}$. Figure E.5 applies cognitive discounting separately to the part of $\mathbf{M}$ that comes from income and taxes. Panel (a) solves for $\left(\mathbf{I} - \mathbf{M}\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^\delta d\mathbf{T}$: agents cognitively discount taxes but understand the boom perfectly. Panel (b) $\left(\mathbf{I} - \mathbf{M}^\delta\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T}$: agents anticipate taxes perfectly, but cognitively discount the endogenous boom. We see that the former effect is expansionary, while the latter is contractionary, as claimed in the main text.

[A-29] Intuitively, $F_{\cdot,s}$ is the impulse response to knowledge at date 0 that a shock will take place at date $s$, and cognitive discounting discounts this at rate $\delta$ with the horizon $s$.

Figure E.5: Cumulative multiplier with cognitive discounting separately applied to taxes and income




Note: calibration as in panel (a) of figure 6. Panel (a) only applies cognitive discounting to taxes, solving $(\mathbf{I}-\mathbf{M})\,d\mathbf{Y} = d\mathbf{G}-\mathbf{M}^{\delta}d\mathbf{T}$. Panel (b) only applies cognitive discounting to output, solving $\left(\mathbf{I}-\mathbf{M}^{\delta}\right)d\mathbf{Y} = d\mathbf{G}-\mathbf{M}d\mathbf{T}$.

# F Appendix to section 6

## F.1 Allowing for valuation effects

In the environment of section 6.1, households receive post-tax labor income $z_{it} = \tau_t\left(w_t e_{it} N_t\right)^{1-\theta} = Z_t e_{it}^{1-\theta} / \int e_{it}^{1-\theta} di$ at time $t$, where $Z_t = w_t N_t - T_t$, and can purchase two types of assets at time $t$: real government bonds, which promise deliver the real interest rate $r_{t+1}$ in period $t+1$, and firm shares, price $p_t$ and dividends $d_t$ at time $t$. These assets can be purchased in any of the accounts that they have. To economize on notation, we spell out the cases of the structural models we use in the paper.

**Single-account models.** In the analytical models of section 4.1 (RA, TA, BU, TABU) and the one-account models of section 4.2 (HA-one, HA-hi-liq, ZL), unconstrained households have share holdings $v_{it-1}$ and bond holdings $b_{it-1}$ in their single account coming into period $t$. They maximize their utility function (26) subject to budget constraint and borrowing constraint,

$$\begin{aligned} c_{it} + p_t v_{it} + b_{it} &= z_{it} + (p_t + d_t)\, v_{it-1} + (1 + r_{t-1})\, b_{i,t-1} \\ p_t v_{it} + b_{it} &\geq 0 \end{aligned} \tag{A.113}$$

Because at least some agents can freely trade bonds for shares, the no arbitrage condition (32) must hold at all $t \geq 0$. Let us consolidate their holdings into an overall asset position, $a_{it} \equiv p_t v_{it} + b_{it}$. Given the no arbitrage condition, at $t \geq 1$ we have $p_t + d_t = (1 + r_{t-1})\, p_{t-1}$, so this requires simply

$$\begin{aligned} c_{it} + a_{it} &= z_{it} + (1 + r_{t-1})\, a_{it-1} \\ a_{it} &\geq 0 \end{aligned}$$

which is the standard borrowing constraint and budget constraint (9) of the one-account model. At date 0, instead, we have only:

$$c_{i0} + a_{i0} = z_{i0} + (p_0 + d_0)\, v_{i-1} + (1 + r_{-1})\, b_{i,-1}$$

We define $\varpi_{i,-1} \equiv \frac{p_{-1} v_{i-1}}{a_{i-1}}$ as the fraction of date-$-1$ assets held in stocks. This then rewrites as:

$$c_{i0} + a_{i0} = z_{i0} + \left( \left( \frac{p_0 + d_0}{p_{-1}} \right) \varpi_{i,-1} + (1 + r_{-1})\,(1 - \varpi_{i,-1}) \right) a_{i-1} \tag{A.114}$$

Note that since there a unit mass of shares outstanding, market clearing for shares in period $-1$ implies that $\int \varpi_{i,-1} a_{i-1} di = p_{-1} \int v_{i-1} di = p_{-1}$, as stated in the main text. The Euler equation for this problem is (A.87).

**Two-account model.** In the two account model of section 4.3 (HA-two), households hold shares $v^{liq}_{it-1}, v^{illiq}_{it-1}$ in their liquid and illiquid account, and bonds $b^{liq}_{it-1}, b^{illiq}_{it-1}$ in these accounts. Their budget and borrowing constraints are now:

$$\begin{aligned} \widetilde{c}_{it} + p_t v^{liq}_{it} + b^{liq}_{it} &= z_{it} + (1 - \zeta) \left\{ (p_t + d_t)\, v^{liq}_{it-1} + (1 + r_{t-1})\, b^{liq}_{it-1} \right\} - d_{it} \cdot 1_{\{adj_{it} = 1\}} \\ p_t v^{illiq}_{it} + b^{illiq}_{it} &= (p_t + d_t)\, v^{illiq}_{it-1} + (1 + r_{t-1})\, b^{illiq}_{it-1} + d_{it} \cdot 1_{\{adj_{it} = 1\}} \\ p_t v^{liq}_{it} + b^{liq}_{it} &\geq 0, \quad p_t v^{illiq}_{it} + b^{illiq}_{it} \geq 0 \end{aligned} \tag{A.115}$$

This says the following: when assets are in the liquid account, the bank levies an intermediation fee $\zeta$ proportional to the value of the assets in that account every period. Having paid the intermediation fee, households are free to exchange assets within this account. When assets are in the illiquid account, there is no intermediation fee, and trading is free. The borrowing constraints on each account depend on the value of the account. However, trading in and out of the illiquid account requires getting the opportunity to rebalance $adj_{it}$, which occurs iid with probability $\nu$.

Following the same steps as above, we consolidate $a^{liq}_{it} = p_t v^{liq}_{it} + b^{liq}_{it}$ and $a^{illiq}_{it} = p_t v^{illiq}_{it} + b^{illiq}_{it}$. For all $t \geq 0$, this gives:

$$\widetilde{c}_{it} + a^{liq}_{it} = z_{it} + (1 - \zeta)\,(1 + r_{t-1})\, a^{liq}_{it-1} - d_{it} \cdot 1_{\{adj_{it} = 1\}} \tag{A.116}$$

$$a^{illiq}_{it} = (1 + r_{t-1})\, a^{illiq}_{it-1} + d_{it} \cdot 1_{\{adj_{it} = 1\}} \tag{A.117}$$

$$a^{liq}_{it} \geq 0, \quad a^{illiq}_{it} \geq 0$$

which are the standard equations for the two-account model with time-varying interest rates. At

$t = 0$, we obtain:

$$\widetilde{c}_{i0} + a_{i0}^{liq} = z_{i0} + (1-\zeta)\left\{\left(\frac{p_0+d_0}{p_{-1}}\right)\omega_{i,-1}^{liq} + (1+r_{-1})\left(1-\omega_{i,-1}^{liq}\right)\right\} a_{i-1}^{liq} - d_{i0}\cdot 1_{\{adj_{it}=1\}} \tag{A.118}$$

$$a_{i0}^{illiq} = \left(\left(\frac{p_0+d_0}{p_{-1}}\right)\omega_{i,-1}^{illiq} + (1+r_{-1})\left(1-\omega_{i,-1}^{illiq}\right)\right) a_{i-1}^{illiq} + d_{i0}\cdot 1_{\{adj_{it}=1\}} \tag{A.119}$$

where we have defined $\omega_{i,-1}^{liq} \equiv \frac{p_{-1}v_{i-1}^{liq}}{a_{i-1}^{liq}}$ and $\omega_{i,-1}^{illiq} = \frac{p_{-1}v_{i-1}^{illiq}}{a_{i-1}^{illiq}}$ as the fraction of the liquid and illiquid accounts invested in shares, respectively. Here, we have $\int \left(\omega_{i,-1}^{liq}a_{i-1}^{liq} + \omega_{i,-1}^{illiq}a_{i-1}^{liq}\right) di = p_{-1}$. The first-order optimality conditions for this problem are (A.101)–(A.105).

**Aggregate consumption function with valuation effects.** We start from a steady state where $r_{-1} = r^{ss}$, $p_{-1} = p^{ss}$, and the distribution over $(e, a_{i-1})$ or $\left(e, a_{i-1}^{liq}, a_{i-1}^{illiq}\right)$ is the stationary distribution. For given distribution of $\omega_{i,-1}$ in the one account model, or $\left(\omega_{i,-1}^{liq}, \omega_{i,-1}^{illiq}\right)$ in the two-account model, the equations above show that the aggregate behavior of the household sector is summarized by the time paths $\{Z_s, r_s\}$ and the initial, cum-dividend stock market price $p_0 + d_0$. We can therefore apply the results from section A.1 to show that the consumption function is:

$$\mathcal{C}_t\left(\{Z_s, r_s\}; p_0 + d_0\right)$$

This is equation (33).

## F.2 Proof of proposition 8

The proof of the proposition is as follows. We will show that equation (35) holds when applied to every basis vector $\mathbf{e}_s$, ie the vector with 0 everywhere, except at $s$ where it has a 1. To do this, we define $\mathbf{U} \equiv \sum_{k=0}^{\infty} \mathbf{F}^k$ as the matrix with ones on and above the main diagonal and zeros below. We then prove the following lemma:

**Lemma 7.** *For the RA, TA, HA-one, ZL, and HA-two models with $\sigma = 1$, and equal initial portfolio shares $\omega_{i,-1} = \omega$ (with $\omega_{i,-1}^{liq} = \omega_{i,-1}^{illiq} = \omega$ in HA-two), in response to a shock $\{d\mathbf{r}_s, d\mathbf{Z}_s, dcap_{0,s}\}$, with*

$$d\mathbf{r}_s \equiv -\mathbf{e}_s; \qquad d\mathbf{Z}_s \equiv Z\mathbf{U}\mathbf{e}_s; \qquad dcap_{0,s} \equiv (1+r)\, A\mathbf{1}'\mathbf{e}_s$$

*aggregate household behavior implies $d\mathbf{C}_s = C\mathbf{U}\mathbf{e}_s$, for any $s$.*

The idea is similar to Werning (2015): given these changes in perceived interest rates, aggregate income and capital gains, and given that individual income is distributed in proportion to aggregate income, and that individual capital gains are proportional to aggregate capital gains in each account, aggregate consumption moves as if the Euler equation applied to every agent.

This requires an EIS $\sigma = 1$, and in the two-account model relies on our assumptions about the distribution of income and capital gains.

Applying (34) to the shocks and outcome behavior in lemma 7, we obtain:

$$\begin{aligned} C\mathbf{U}\mathbf{e}_s &= \mathbf{M}d\mathbf{Z}_s + \mathbf{M}^r d\mathbf{r}_s + \mathbf{m}^{cap} dcap_{0,s} \\ &= \mathbf{M}Z\mathbf{U}\mathbf{e}_s - \mathbf{M}^r\mathbf{e}_s + (1+r)\, A\mathbf{m}^{cap}\mathbf{1}'\mathbf{e}_s \end{aligned}$$

Substituting $Z = C - rA$ and rearranging, this implies

$$\begin{aligned} \mathbf{M}^r\mathbf{e}_s &= \left(\mathbf{M}Z\mathbf{U} - C\mathbf{U} + (1+r)\, A\mathbf{m}^{cap}\mathbf{1}'\right)\mathbf{e}_s \\ &= \left(-C\mathbf{U}\left(\mathbf{I} - \left(1 - r\frac{A}{C}\right)\mathbf{M}\right) + (1+r)\, A\mathbf{m}^{cap}\mathbf{1}'\right)\mathbf{e}_s \end{aligned}$$

and since this relationship holds for all $s$, we obtain (35), completing the proof of the proposition.

To prove lemma 7, we note that for any $s$, our proposed shock $\{d\mathbf{r}_s, d\mathbf{Z}_s, dcap_{0,s}\}$ can be written explicitly, dropping $s$ subscripts for ease of notation, as:

$$d\log(1+r_t) = -1_{t=s} \tag{A.120}$$

$$dZ_t = Z \cdot 1_{t\leq s} \tag{A.121}$$

$$d\text{cap}_0 = (1+r)A \tag{A.122}$$

This implies in particular, for individual $i$, that $dz_{it} = \frac{z_{it}}{Z} \cdot dZ_t = z_{it} \cdot 1_{t\leq s}$.

*Proof of lemma 7 for homothetic one-account models (RA, TA, HA-one and ZL).* For these models, we show that the shock (A.120)–(A.122) results in changes to consumption and assets at the individual level of:

$$dc_{it} = c_{it} \cdot 1_{t\leq s} \tag{A.123}$$

$$da_{it} = a_{it} \cdot 1_{t\leq s} \tag{A.124}$$

We will use the notation for HA-one, with ZL being nested as a limit of HA-one, and RA being nested as the special case with only one agent and no idiosyncratic uncertainty. For TA, the conjecture holds for the permanent-income agent like with RA, and it follows trivially from $dc_{it} = dz_{it}$ and $da_{it} = a_{it} = 0$ for the hand-to-mouth agent.

*Euler equation.* First, taking the Euler equation (A.87) when it holds with equality, with $u(c) = \log c$, we have:

$$\frac{1}{c_{it}} = \beta(1+r_t)\mathbb{E}_t\left[\frac{1}{c_{it+1}}\right] \tag{A.125}$$

Totally differentiating, we obtain:

$$\frac{dc_{it}}{c_{it}^2} = \beta(1+r)\mathbb{E}_t\left[\frac{dc_{it+1}}{c_{it+1}^2}\right] - \beta dr_t \mathbb{E}_t\left[\frac{1}{c_{it+1}}\right] \tag{A.126}$$

We check that our guess (A.120)–(A.124) satisfies (A.126). For $t < s$, we have $dr_t = 0$, $dc_{it} = c_{it}$, and $dc_{it+1} = c_{it+1}$, and (A.126) reduces to the Euler equation (A.125). For $t > s$, both sides of (A.126) are identically zero. For $t = s$, we have $-dr_t = 1+r$, $dc_{it} = c_{it}$, and $dc_{it+1} = 0$, and (A.126) again reduces to the Euler equation (A.125).

When the Euler equation initially holds with inequality, then it continues to do so under a small perturbation.

*Budget constraint at $t > 0$.* Here, the budget constraint is simply (9),

$$c_{it} + a_{it} = z_{it} + (1+r_{t-1})a_{it-1}$$

Totally differentiating and using $dz_{it} = \frac{z_{it}}{Z} \cdot dZ_t$, we get

$$dc_{it} + da_{it} = \frac{z_{it}}{Z} dZ_t + (1+r)da_{it-1} + dr_{t-1}a_{it-1} \tag{A.127}$$

For $t < s+1$, we have $dr_{t-1} = 0$, $(1+r)da_{it-1} = a_{it-1}$, $\frac{z_{it}}{Z}dZ_t = z_{it}$, and $dc_{it} + da_{it} = c_{it} + a_{it}$, so (A.127) reduces to the budget constraint (9). For $t > s+1$, both sides of (A.127) are identically zero. For $t = s+1$, $(1+r)da_{it-1} = (1+r)a_{it-1}$ and $dr_{t-1}a_{it-1} = -(1+r)a_{it-1}$, cancelling out, with all other terms in (A.127) being zero. This is the step at which assuming an EIS $\sigma$ of 1 is critical. If, for instance, we had a higher $\sigma$, then $dc_{it}$ and $da_{it}$ would have needed to be larger given $dr_s$, and then these two terms would not cancel.

*Budget constraint at $t = 0$.* Here, the date-0 budget constraint is (A.114). Enforcing our assumption that $\varpi_{i,-1} = \varpi$ for all agents, this gives:

$$c_{i0} + a_{i0} = z_{i0} + \frac{p_0 + d_0}{p_{-1}}\varpi a_{i,-1} + (1+r_{-1})(1-\varpi)a_{i,-1}$$

Totally differentiating, using the definition $d\text{cap}_0 = d(p_0 + d_0)$, this becomes

$$dc_{i0} + da_{i0} = dz_{i0} + \frac{d\text{cap}_0}{p_{-1}}\varpi a_{i,-1} \tag{A.128}$$

Now, since we have $p_{-1} = \int \varpi_{i,-1}a_{i-1}di = \varpi A$. Using our guess (A.122), this implies $\frac{d\text{cap}_0}{p_{-1}} = (1+r)\frac{A}{p_{-1}} = (1+r)\frac{1}{\varpi}$, and therefore $\varpi a_{i,-1}\frac{d\text{cap}_0}{p_{-1}} = \omega a_{i,-1}(1+r)\frac{1}{\varpi} = (1+r)a_{i,-1}$. Since $dz_{i0} = z_{i0}$, $dc_{i0} = c_{i0}$, and $da_{i0} = a_{i0}$, (A.128) then reduces to the steady-state budget constraint (9) for $t = 0$. This complete the proof of lemma for our homothetic one-account models. □

*Proof of lemma 7 for two-account model (HA-two).* For our HA-two model, we similarly conjecture that:

$$d\tilde{c}_{it} = \tilde{c}_{it} \cdot 1_{t\leq s}$$
$$da^{liq}_{it} = a^{liq}_{it} \cdot 1_{t\leq s}$$
$$da^{illiq}_{it} = a^{illiq}_{it} \cdot 1_{t\leq s}$$
$$d(d_{it}) = d_{it} \cdot 1_{t\leq s}$$
$$d(V_{a^{illiq},t}) = -V_{a^{illiq},t} \cdot 1_{t\leq s+1}$$

As above, we verify the Euler equation, as well as the budget constraints separately in the $t > 0$ and $t = 0$ cases.

*Euler equations.* Consolidating (A.101)–(A.105) by solving out for $V_{a^{liq}}$ and using the $\sigma = 1$ assumption, the solution for agents that are not on a constraint is characterized by the four conditions

$$\frac{1}{\tilde{c}_{it}} = \beta(1+r_t)(1-\zeta)\mathbb{E}\left[\frac{1}{\tilde{c}_{it+1}}\right] \tag{A.129}$$
$$\frac{1}{\tilde{c}_{it}} = \beta\mathbb{E}\left[V_{a^{illiq},i,t+1}\right] \qquad \text{if } adj_{it} = 1 \tag{A.130}$$
$$V_{a^{illiq},i,t} = (1+r_{t-1})\frac{1}{\tilde{c}_{it}} \qquad \text{if } adj_{it} = 1 \tag{A.131}$$
$$V_{a^{illiq},i,t} = \beta(1+r_t)\mathbb{E}\left[V_{a^{illiq},i,t+1}\right] \qquad \text{if } adj_{it} = 0 \tag{A.132}$$

The argument for why our guess satisfies the (A.129) is the same as in the HA-one case. Totally differentiating (A.130), we find, if $adj_{it} = 1$,

$$\frac{-d\tilde{c}_{it}}{\tilde{c}^2_{it}} = \beta\mathbb{E}\left[dV_{a^{illiq},i,t+1}\right]$$

which, given our guess, reduces to (A.130) for $t \leq s$, and is identically zero for $t > s$. Totally differentiating (A.131), we obtain, when $adj_{it} = 1$,

$$dV_{a^{illiq},i,t} = dr_{t-1}\frac{1}{\tilde{c}_{it}} - (1+r)\frac{d\tilde{c}_{it}}{\tilde{c}^2_{it}}$$

When $t \leq s$, we have $dr_{t-1} = 0$, so the left-hand side is $-V_{a^{illiq},t}$ and the right-hand side is $-\frac{1+r}{\tilde{c}_{it}}$, reducing to (A.131). When $t = s+1$, we have $dr_{t-1} = -(1+r)$ and $d\tilde{c}_{it} = 0$, so the right-hand side is also $-\frac{1+r}{\tilde{c}_{it}}$ and the equation again reduces to (A.131). When $t > s+1$, both sides are identically zero. Totally differentiating (A.132), we obtain, when $adj_{it} = 0$,

$$dV_{a^{illiq},i,t} = \beta dr_{t-1}\mathbb{E}\left[V_{a^{illiq},i,t+1}\right] + \beta(1+r)\mathbb{E}\left[V_{a^{illiq},i,t+1}\right]$$

This again reduces to (A.132) for $t \leq s$ due to the $\mathbb{E}\left[V_{a^{illiq},i,t+1}\right]$ term, and for $t = s+1$ due to the $dr_{t-1}$ term; while it is identically zero for $t > s+1$. Finally, whenever constraints bind, they continue to do so under a small perturbation. This verifies our conjecture that all Euler equations continue to hold in our perturbation.

*Budget constraint at $t > 0$.* Here, the liquid account budget constraint is (A.116),

$$\widetilde{c}_{it} + a_{it}^{liq} = z_{it} + (1 + r_{t-1})(1-\zeta)a_{it-1}^{liq} - d_{it} \cdot 1_{adj_{it}=1}$$

The proof is the same as in the HA-one case, with everything scaling proportionally with the shock for $t < s+1$, everything unchanged for $t > s+1$, and everything except $(1 + r_{t-1})(1-\zeta)a_{it-1}^{liq}$ obviously unchanged for $t = s+1$, and then offsetting effects so that $(1 + r_{t-1})(1-\zeta)a_{it-1}^{liq}$ also does not change to first order. The exact same argument applies to the illiquid account budget constraint (A.117).

*Budget constraint at $t = 0$.* Here, the liquid account budget constraint is (A.118). Enforcing our assumption that $\varpi_{i,-1}^{liq} = \varpi$, this is

$$\widetilde{c}_{i0} + a_{i0} = z_{i0} + \frac{p_0 + d_0}{p_{-1}}\varpi a_{i,-1}^{liq} + (1+r)(1-\varpi)a_{i,-1}^{liq} - d_{i0} \cdot 1_{adj_{i0}=1}$$

and analogous to the HA-one case, totally differentiating, this becomes

$$d\widetilde{c}_{i0} + da_{i0} = dz_{i0} + \frac{d\text{cap}_0}{p_{-1}}\varpi a_{i,-1}^{liq} - d(d_{i0}) \cdot 1_{adj_{i0}=1} \tag{A.133}$$

where $\frac{d\text{cap}_0}{p_{-1}}\varpi a_{i,-1}^{liq} = (1+r)a_{i,-1}^{liq}$ and all other variables also change proportionally, so that the budget constraint continues to hold. The illiquid account budget constraint, (A.119), enforcing $\varpi_{i,-1}^{illiq} = \varpi$

$$a_{i0}^{illiq} = \frac{p_0 + d_0}{p_{-1}}\varpi a_{i,-1}^{illiq} + (1+r)(1-\varpi)a_{i,-1}^{illiq} + d_{i0} \cdot 1_{adj_{i0}=1}$$

and we verify again that this reduces to (A.119) under our guess. Note that all accounts hold the same fraction in shares is important to ensure that the shock does not alter the portfolio allocation between the liquid and the illiquid account. This completes the proof. □

### F.3 Relaxing $\sigma = 1$

To accommodate the case where the elasticity of intertemporal substitution $\sigma$ is different from 1, we generalize the budget constraint so that in the one-account model it reads

$$c_{it} + a_{it} = z_{it} + (1 + r_{t-1})(1-\hat{\omega})a_{i,t-1} + \frac{p_t + d_t}{p_{t-1}}\hat{\omega}a_{i,t-1} \tag{A.134}$$

where we now allow for $(p_t + d_t)/p_{t-1}$ to differ from $1 + r_{t-1}$ away from the steady state by some perturbation $dcap_t \equiv d((p_t + d_t) - (1 + r_{t-1})p_{t-1})$, which generalizes $dcap_0$. We assume that the

Euler equation still depends only on $r_t$ and is unaffected by the additional asset return $dcap_t$.[A-30] We also extend the assumption of constant portfolio shares $\hat{\omega}$ to all periods.

We make analogous modifications to the budget constraints in all other models, and then define the matrix $\mathbf{M}^{cap}$ to be the Jacobian of consumption with respect to the vector $d\mathbf{cap}$, which stacks the $dcap_t$. We then have the following generalization of proposition 8.

**Proposition 12.** *For the RA, TA, HA-one, ZL, and HA-two models, with any elasticity of intertemporal substitution $\sigma$, and equal portfolio shares, we have:*

$$\mathbf{M}^r = -\sigma C \left( \mathbf{I} - \left(1 - \frac{rA}{C}\right) \mathbf{M} \right) \mathbf{U} + \sigma(1+r)A\mathbf{m}^{cap}\mathbf{1}' + (1-\sigma)(1+r)A\mathbf{M}^{cap}\mathbf{L} \tag{A.135}$$

*where the matrix $\mathbf{M}^{cap}$ is the Jacobian of consumption with respect to asset return shocks $d\mathbf{cap}$.*

Note that (A.135) is mostly similar to the $\sigma = 1$ case (35). Intuitively, since we expect the response to interest rates to scale with the intertemporal elasticity of substitution, it multiplies the two terms from (35) by $\sigma$. There is, however, an additional term $(1-\sigma)(1+r)A\mathbf{M}^{cap}\mathbf{L}$. This is a correction to reflect the fact that the direct income effects of real interest rates do not scale with $\sigma$.

The proof is quite similar to that of proposition 8, and we will proceed analogously, skipping some steps to conserve space. First, we have the lemma

**Lemma 8.** *For the RA, TA, HA-one, ZL, and HA-two models with any elasticity of intertemporal substitution $\sigma$, and equal portfolio shares, in response to a shock $\{d\mathbf{r}_s, d\mathbf{Z}_s, d\mathbf{cap}_s\}$, with*

$$d\mathbf{r}_s \equiv -\mathbf{e}_s; \qquad d\mathbf{Z}_s \equiv \sigma Z \mathbf{U}\mathbf{e}_s; \qquad d\mathbf{cap}_s \equiv (1+r)\, A(\sigma\mathbf{e}_0 + (1-\sigma)\mathbf{L}\mathbf{e}_s)$$

*aggregate household behavior implies $d\mathbf{C}_s = \sigma C\mathbf{U}\mathbf{e}_s$, for any $s$.*

Totally differentiating consumption in response to the shocks in lemma 8, we obtain

$$\begin{aligned} \sigma C\mathbf{U}\mathbf{e}_s &= \mathbf{M}d\mathbf{Z}_s + \mathbf{M}^r d\mathbf{r}_s + \mathbf{M}^{cap} d\mathbf{cap}_s \\ &= \sigma\mathbf{M}Z\mathbf{U}\mathbf{e}_s - \mathbf{M}^r\mathbf{e}_s + (1+r)A(\sigma\mathbf{M}^{cap}\mathbf{e}_0 + (1-\sigma)\mathbf{M}^{cap}\mathbf{L}\mathbf{e}_s) \end{aligned}$$

Substituting $Z = C - rA$ and $\mathbf{M}^{cap}\mathbf{e}_0 = \mathbf{m}^{cap}$ (since the 0th column of $\mathbf{M}^{cap}$ is just our original $\mathbf{m}^{cap}$) and rearranging, this becomes

$$\begin{aligned} \mathbf{M}^r\mathbf{e}_s &= \left(\sigma\mathbf{M}Z\mathbf{U} - \sigma C\mathbf{U} + \sigma(1+r)A\mathbf{m}^{cap}\mathbf{1}' + (1-\sigma)(1+r)A\mathbf{M}^{cap}\mathbf{L}\right)\mathbf{e}_s \\ &= \left( -\sigma C\mathbf{U}\left(\mathbf{I} - \left(1 - r\frac{A}{C}\right)\mathbf{M}\right) + \sigma\,(1+r)\,A\mathbf{m}^{cap}\mathbf{1}' + (1-\sigma)\mathbf{M}^{cap}\mathbf{L} \right)\mathbf{e}_s \end{aligned}$$

and since this holds for each $s$, (A.135) follows.

[A-30] We can also interpret the response to $dcap_t$ as the income effect of a change in rates from $r_{t-1}$ to $r_t$, suppressing the substitution effect that works through the Euler equation.

*Proof of lemma 8.* Similar to the proof of lemma 7, we write our proposed shock explicitly for a given $s$,

$$\begin{aligned} d\log(1+r_t) &= -1_{t=s} \\ dZ_t &= \sigma Z \cdot 1_{t\leq s} \\ dcap_0 &= \sigma(1+r)A \\ dcap_{t+1} &= (1-\sigma)(1+r)A \end{aligned}$$

We then guess and verify that this is consistent with a policy $dc_{it} = \sigma c_{it} \cdot 1_{t\leq s}$ and $da_{it} = \sigma a_{it} \cdot 1_{t\leq s}$ for the HA-one model. The extension to other models is similar to lemma 7, and we omit it to conserve space.

*Euler equation.* First, the Euler equation when binding is

$$c_{it}^{-\sigma^{-1}} = \beta(1+r_t)\mathbb{E}_t c_{it+1}^{-\sigma^{-1}} \tag{A.136}$$

Totally differentiating gives

$$\sigma^{-1} dc_{it} \cdot c_{it}^{-\sigma^{-1}-1} = \sigma^{-1}\beta(1+r)\mathbb{E}_t dc_{it+1} c_{it+1}^{-\sigma^{-1}-1} - \beta dr_t \mathbb{E}_t c_{it+1}^{-\sigma^{-1}} \tag{A.137}$$

For $t < s$, we have $dr_t = 0$, $dc_{it} = \sigma c_{it}$, and $dc_{it+1} = \sigma c_{it+1}$. Substituting these into (A.137) gives the Euler equation (A.136). For $t > s$, (A.137) is identically zero on both sides. For $t = s$, we have $dc_{it} = \sigma c_{it}$ and $-dr_t = 1 + r$, so that (A.137) again reduces to the Euler equation (A.136). When not binding, the Euler equation as before is not affected by an infinitesimal perturbation. □

*Budget constraint.* The modified budget constraint (A.134) from above is

$$c_{it} + a_{it} = z_{it} + (1+r_{t-1})(1-\hat{\omega})a_{i,t-1} + \frac{p_t + d_t}{p_{t-1}}\hat{\omega} a_{i,t-1}$$

Using the definition $dcap_t = d((p_t + d_t) - (1 + r_{t-1})p_{t-1})$, we have

$$d\frac{p_t + d_t}{p_{t-1}} = dr_{t-1} + \frac{dcap_t}{p}$$

Now, using this to totally differentiate the entire budget constraint, and noting that the ratio of the portfolio share $\hat{\omega}$ to steady-state $p$ is $\frac{1}{A}$, we have

$$dc_{it} + da_{it} = dz_{it} + dr_{t-1}a_{i,t-1} + (1+r)da_{i,t-1} + dcap_t \frac{1}{A} a_{i,t-1} \tag{A.138}$$

Recall that $dz_{it} = \frac{z_{it}}{Z} dZ_t = \sigma z_{it} \cdot 1_{t\leq s}$ here.

For $t > s+1$, everything is identically zero. For $0 < t < s+1$, we have $dcap_t = 0$ and $dr_{t-1} = 0$. In these cases, (A.138) reduces to $dc_{it} + da_{it} = dz_{it} + (1+r)da_{i,t-1}$, which becomes $\sigma c_{it} + \sigma a_{it} = \sigma z_{it} + (1+r)\sigma a_{i,t-1}$, just the budget constraint times $t$.

For $t = 0$, (A.138) becomes

$$dc_{i0} + da_{i0} = dz_{i0} + dcap_0 \frac{1}{A} a_{i,-1}$$

which becomes $\sigma c_{i0} + \sigma a_{i0} = \sigma z_{i0} + \frac{(1+r)A}{A} a_{i,-1} = \sigma z_{i0} + (1+r) a_{i,-1}$, which is again the budget constraint. Finally, for $t = s+1$, (A.138) becomes

$$0 = -(1+r)a_{i,s} + (1+r)\sigma a_{i,s} + (1-\sigma)\frac{(1+r)A}{A} a_{i,s}$$

which indeed evaluates to zero.

### F.4 Relation between $\mathbf{M}$ and $\mathbf{M}^r$ in TABU model

In the BU model, we have the following proposition.

**Proposition 13.** *Consider the BU model with elasticity of intertemporal substitution $\sigma = 1$. Let $\psi \equiv -\chi'(A)/(\chi''(A)A)$ denote the equilibrium inverse curvature in utility from assets. Then we have:*

$$\mathbf{M}^r = -C\left(\mathbf{I} - \left(1 - \psi\frac{rA}{C}\right)\mathbf{M}\right)\mathbf{U} + \psi(1+r)A\mathbf{m}^{cap}\mathbf{1}' + (1-\psi)(1+r)A\mathbf{M}\mathbf{L} \tag{A.139}$$

Again, the proof of this proposition follows the same argument as that of proposition 8 in section F.2, except that the perturbation is different. We have the following modification of lemma 7.

**Lemma 9.** *For the BU model with $\sigma = 1$, in response to a shock $\{d\mathbf{r}_s, d\mathbf{Z}_s, dcap_{0,s}\}$, with*

$$d\mathbf{r}_s \equiv -\mathbf{e}_s; \qquad d\mathbf{Z}_s \equiv \left((Z + (1-\psi)rA)\cdot\mathbf{U} + (1-\psi)(1+r)A\cdot\mathbf{L}\right)\mathbf{e}_s; \qquad dcap_{0,s} \equiv \psi\,(1+r)\,A\mathbf{1}'\mathbf{e}_s$$

*aggregate household behavior implies $d\mathbf{C}_s = C\mathbf{U}\mathbf{e}_s$ and $d\mathbf{A}_s = \psi A\mathbf{U}\mathbf{e}_s$, for any $s$.*

This perturbation corrects for the way in which the income effect accrues in the BU model. Applying (34) to these shocks and outcome behavior, and noting that this holds for all $\mathbf{e}_s$ and therefore holds in matrix form, we have

$$C\mathbf{U} \;=\; \mathbf{M}\left((Z + (1-\psi)rA)\cdot\mathbf{U} + (1-\psi)(1+r)A\cdot\mathbf{L}\right) - \mathbf{M}^r + \psi\,(1+r)\,A\mathbf{m}^{cap}\mathbf{1}'$$

Writing $Z + (1-\psi)rA = C - \psi rA$, and rearranging, this delivers equation (A.139), which makes three changes relative to (35) in Proposition 8: it dampens $A$ by $\psi$ in its two appearances in (35), and then adds the term $(1-\psi)(1+r)A\mathbf{M}\mathbf{L}$.

*Proof of lemma 9.* For given $s$, the perturbation in lemma 9 can be written explicitly for each $t$ (dropping the $s$ subscript for simplicity), as:

$$d\log(1+r_t) = -1_{t=s}$$
$$dZ_t = (Z + (1-\psi)rA)\cdot 1_{t\le s} + (1-\psi)(1+r)A\cdot 1_{t=s+1}$$
$$d\text{cap}_0 = \psi(1+r)A$$

We conjecture that $dC_t = C\cdot 1_{t\le s}$ and $dA_t = \psi A\cdot 1_{t\le s}$. We again show that the Euler equation and budget constraints at $t>0$ and $t=0$ are satisfied.

*Euler equation.* First, the Euler equation is (A.55), where the interest rate can now vary. With $\sigma$ =1 this reads:

$$\frac{1}{C_t} = \beta(1+r_t)\frac{1}{C_{t+1}} + \chi'(A_t) \tag{A.140}$$

Totally differentiated, this becomes

$$-\frac{dC_t}{C^2} = -\beta(1+r)\frac{dC_{t+1}}{C^2} + \beta dr_t\frac{1}{C} + \chi''(A)dA_t$$

or equivalently, given our definition of $\psi \equiv -\chi'(A)/\left(\chi''(A)A\right)$,

$$-\frac{dC_t}{C^2} = -\beta(1+r)\frac{dC_{t+1}}{C^2} + \beta dr_t\frac{1}{C} - \chi'(A)\frac{1}{\psi}\frac{dA_t}{A}$$

For $t<s$, $dC_t = C$, $dA_t/\psi = A$, $dC_{t+1} = C$, and $dr_t = 0$, and this reduces to the steady-state Euler equation (A.140). For $t>s$, all terms are identically zero. For $t=s$, $dC_t$ and $dA_t$ are unchanged, but $dC_{t+1} = 0$ and $dr_t = -(1+r)$, so that $\beta dr_t\frac{1}{C} = -\beta(1+r)\frac{1}{C}$ and again this reduces to the steady-state Euler (A.140).

*$t>0$ budget constraint.* The budget constraint at date $t>0$ is (9), which given that there is a single BU agent is simply

$$C_t + A_t = (1+r_{t-1})A_{t-1} + Z_t$$

and totally differentiated we can write this as

$$dC_t + dA_t - (1+r)dA_{t-1} - dr_{t-1}A = dZ_t$$

For $0<t<s+1$, we have $dC_t = C$, $dA_t = dA_{t-1} = \psi A$, and $dr_{t-1} = 0$, so that the left side is just $C - \psi rA$, which from $C = Z + rA$ can be rewritten as $Z + (1-\psi)rA$, which equals $dZ_t$. For $t > s+1$, this is identically zero. Finally, for $t = s+1$, we have $dC_t = dA_t = 0$, $(1+r)dA_{t-1} = \psi(1+r)A$, and $dr_{t-1}A = -(1+r)A$, so that the left side equals $(1-\psi)(1+r)A$, which equals $dZ_{s+1}$.

*$t=0$ budget constraint.* The budget constraint at $t=0$ is (A.114), which given that there is a

single BU agent with stock portfolio share $\varpi$ is simply

$$C_0 + A_0 = Z_0 + \frac{p_0 + d_0}{p_{-1}} \varpi A_{-1} + (1+r)(1-\varpi)A_{-1}$$

or totally differentiated

$$dC_0 + dA_0 = dZ_0 + \frac{d\text{cap}_0}{p_{-1}} \omega A$$

Like before, but now with a $\psi$ factor, we can write $\frac{d\text{cap}_0}{p_{-1}} \varpi A = \psi(1+r)A$. Substituting $dC_0$, $dA_0$ and $dZ_0$, this becomes

$$C + \psi A = Z + (1-\psi) r A + \psi(1+r)A$$

and cancelling out the $\psi$ terms leaves $C = Z + rA$, which is just the steady-state budget constraint.

□

*Extension to TABU.* We can extend proposition 13 to the TABU model as follows. We know that $\mathbf{M}^r$ for the hand-to-mouth households is zero, since these households hold no assets and are not on their Euler equations. Hence, we only need to evaluate $\mathbf{M}^r$ for the BU households and multiply by $1-\mu$. Rewriting (A.139), we have:

$$\mathbf{M}^r = -(1-\mu) C^{BU} \left( \mathbf{I} - \left(1 - \psi \frac{rA^{BU}}{C^{BU}}\right) \mathbf{M}^{BU} \right) \mathbf{U} \\ + (1-\mu)\psi(1+r)A^{BU} \mathbf{m}^{cap} \mathbf{1}' + (1-\mu)(1-\psi)(1+r)A^{BU} \mathbf{M}^{BU} \mathbf{L} \quad \text{(A.141)}$$

Since all assets are held by the BU households, $(1-\mu)A^{BU} = A$ and $\mathbf{M}^{BU} = \mathbf{M}^{cap}$, and the second two terms reduce to $\psi(1+r)A\mathbf{m}^{cap}\mathbf{1}' + (1-\psi)(1+r)A\mathbf{M}^{cap}\mathbf{L}$.

We now focus on the coefficient on $\mathbf{U}$ in the first term of (A.141). Multiplying out and expanding $C^{BU} = Z + \frac{rA}{1-\mu}$, this becomes

$$-(1-\mu)C^{BU}\mathbf{I} + (1-\mu)Z\mathbf{M}^{BU} + rA\mathbf{M}^{BU} - \psi r A \mathbf{M}^{BU}$$

Adding $0 = -\mu Z \mathbf{I} + \mu Z \mathbf{I}$, where we note that $C = (1-\mu)C^{BU} + \mu Z$ and $\mathbf{M} = (1-\mu)\mathbf{M}^{BU} + \mu \mathbf{I}$, this simplifies to just

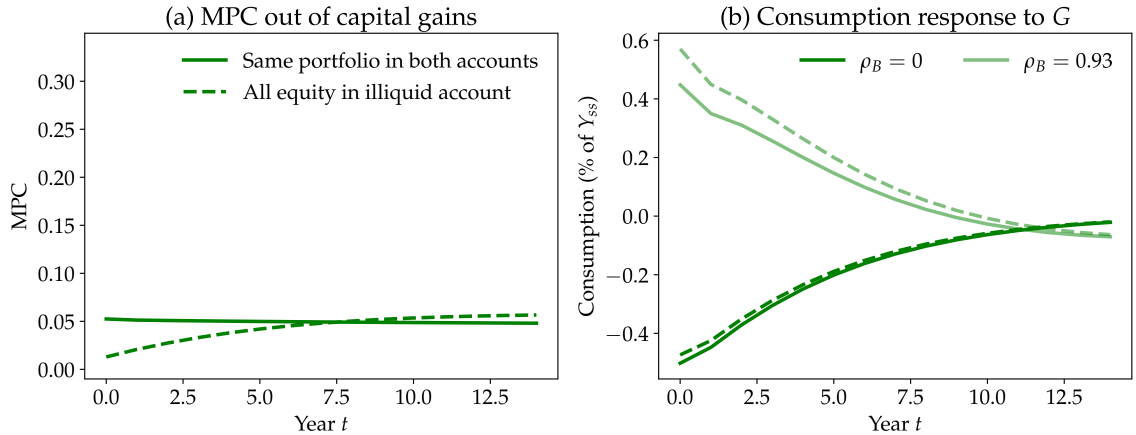
$$-C\mathbf{I} + Z\mathbf{M} + (1-\psi) r A \mathbf{M}^{BU}$$

Combining this with the other two terms derived above, and using $Z = C\left(1 - \frac{rA}{C}\right)$ and $\mathbf{M}^{BU} = \mathbf{M}^{cap}$ again, we summarize our results in the following corollary to proposition 13.

**Corollary 4.** *Consider the TABU model with elasticity of intertemporal substitution $\sigma = 1$. Let $\psi \equiv -\chi'(A)/\left(\chi''(A)A\right)$ denote the equilibrium inverse curvature in utility from assets. Then we have:*

$$\mathbf{M}^r = -C \left( \mathbf{I} - \left(1 - \frac{rA}{C}\right) \mathbf{M} - (1-\psi)\frac{rA}{C} \mathbf{M}^{cap} \right) \mathbf{U} + \psi(1+r)A\mathbf{m}^{cap}\mathbf{1}' + (1-\psi)(1+r)A\mathbf{M}^{cap}\mathbf{L} \quad \text{(A.142)}$$

Figure F.1: Robustness to holding equity only in the illiquid account




### F.5 Role of the common equity share assumption

Applied to the two-account heterogeneous-agent model, proposition 8 assumes that all agents hold equity in their liquid and illiquid accounts in equal proportions. In this section, we relax this assumption and instead assume that all equity is held in illiquid accounts, that is, $\omega_{i,-1}^{liq} = 0$ and $\omega_{i,-1}^{illiq} = \frac{p_{-1}}{A^{illiq}}$. This is possible because, in our calibration, the aggregate steady state illiquid account balance $A^{illiq}$ is smaller than steady state equity $p_{-1}$.

Figure F.1(a) plots the iMPCs out of capital gains, comparing the model with only equity in illiquid accounts with the common equity share model described in section 6. Both models' iMPCs are small; they are even a bit smaller in the model in which all equity is held in illiquid accounts, as capital gains earned in the illiquid account take longer to pass through to consumption.

Figure F.1(b) feeds the same shock $d\mathbf{G}$ as in figure 8 into the quantitative model from section 7; comparing the consumption responses of the model with only equity in illiquid accounts with that of the common equity share model. For both degrees of deficit financing $\rho_B$, the consumption responses are very close. Since the iMPCs out of capital gains are smaller when equity is entirely held in illiquid account, the consumption response is slightly greater in that model.

# G Appendix to section 7

### G.1 Supply side with investment

The equations for the model with investment are as follows. Recall that the production function of each firm is Cobb-Douglas,

$$F(k_{t-1}, n_t) = \Theta k_{t-1}^{\alpha} n_t^{1-\alpha}$$

We first note that the Rotemberg adjustment costs function $\xi\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right) = \frac{1}{2\kappa^p(\mu^p-1)}\left(\frac{\mathcal{P}_t-\mathcal{P}_{t-1}}{\mathcal{P}_{t-1}}\right)^2$ has partial derivatives that satisfy:

$$\xi_p\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right)\mathcal{P}_t = \frac{1}{\kappa^p\left(\mu^p-1\right)}\left(\frac{\mathcal{P}_t-\mathcal{P}_{t-1}}{\mathcal{P}_{t-1}}\right)\frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} \tag{A.143}$$

$$-\xi_{p_-}\left(\mathcal{P}_{t+1}, \mathcal{P}_{t}\right)\mathcal{P}_{t+1} = \frac{1}{\kappa^p\left(\mu^p-1\right)}\left(\frac{\mathcal{P}_{t+1}-\mathcal{P}_{t}}{\mathcal{P}_{t}}\right)\frac{\mathcal{P}_{t+1}}{\mathcal{P}_{t}} \tag{A.144}$$

We also note that the adjustment cost function $\varphi\left(\frac{k_t}{k_{t-1}}\right) \equiv \frac{1}{2\delta\epsilon_I}\left(\frac{k_t}{k_{t-1}}-1\right)^2$ satisfies

$$\varphi'\left(\frac{k_t}{k_{t-1}}\right) = \frac{1}{\delta\epsilon_I}\left(\frac{k_t}{k_{t-1}}-1\right) \tag{A.145}$$

We look for the solution to the firm problem given production constraint $F\left(k_{t-1}, n_t\right) = Y_t\left(\frac{p_t}{P_t}\right)^{-\frac{\mu^p}{\mu^p-1}}$. The firm states are its price $\mathcal{P}_{t-1}$ and its capital stock $k_{t-1}$ from the previous period. The Bellman equation is:

$$\begin{aligned} J_t\left(\mathcal{P}_{t-1}, k_{t-1}\right) &= \max_{\mathcal{P}_t, k_t, n_t}\left\{\frac{\mathcal{P}_t}{P_t}F\left(k_{t-1}, n_t\right) - \frac{W_t}{P_t}n_t - \left(k_t - (1-\delta)\,k_{t-1}\right) - \varphi\left(\frac{k_t}{k_{t-1}}\right)k_{t-1}\right. \\ &\qquad \left. -\xi\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right)Y_t + \frac{1}{1+r_t}J_{t+1}\left(\mathcal{P}_t, k_t\right)\right\} \\ &\text{s.t.} \quad \left(\frac{F\left(k_{t-1}, n_t\right)}{Y_t}\right)^{\frac{1}{\mu^p}-1}Y_t = \frac{\mathcal{P}_t}{P_t}Y_t \end{aligned}$$

Let $\eta_t$ denote the Lagrange multiplier on the production constraint. The first order condition for labor $n_t$ is:

$$\underbrace{\left(\frac{\mathcal{P}_t}{P_t} + \eta_t\left(\frac{1}{\mu^p}-1\right)\left(\frac{F\left(k_{t-1}, n_t\right)}{Y_t}\right)^{\frac{1}{\mu^p}-2}\right)}_{mc_t} F_n\left(k_{t-1}, n_t\right) = \frac{W_t}{P_t} \tag{A.146}$$

Where $mc_t$ denotes marginal cost. In equilibrium all firms chose the same price, so $\mathcal{P}_t = P_t$ and $F\left(k_{t-1}, n_t\right) = Y_t$. This implies the following relationship between marginal cost and the Lagrange multiplier:

$$mc_t = 1 - \eta_t\left(1 - \frac{1}{\mu^p}\right) \tag{A.147}$$

Note that a higher multiplier $\eta_t$ is associated with a lower real marginal cost, with $mc_t \leq 1$, $mc_t = \frac{1}{\mu^p}$ when $\eta_t = 1$ and $mc_t \to 1$ when $\eta_t \to 0$.

The first-order condition for the price $\mathcal{P}_t$ is:

$$\frac{1}{P_t}F\left(k_{t-1}, n_t\right) - \xi_p\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right)Y_t + \frac{1}{1+r_t}\frac{\partial J_{t+1}\left(\mathcal{P}_t, k_t\right)}{\partial \mathcal{P}_t} - \frac{\eta_t}{P_t}Y_t = 0 \tag{A.148}$$

while the envelope condition gives:

$$\frac{\partial J_t\left(\mathcal{P}_{t-1}, k_{t-1}\right)}{\partial \mathcal{P}_{t-1}} = -\xi_{p-}\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right) Y_t \tag{A.149}$$

Combining the two and multiplying by $\mathcal{P}_t$ we obtain

$$\left(\frac{\mathcal{P}_t}{P_t}\left(1-\eta_t\right)\right) Y_t = \xi_p\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right) \mathcal{P}_t Y_t + \frac{1}{1+r_t} \xi_{p-}\left(\mathcal{P}_{t+1}, \mathcal{P}_t\right) \mathcal{P}_t Y_{t+1}$$

This shows that firms indeed choose the same price $\mathcal{P}_t = P_t$. Using (A.143) and (A.144), and the definition of gross inflation $1+\pi_t \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} = \frac{P_t}{P_{t-1}}$, we therefore obtain:

$$\frac{\mathcal{P}_t}{P_t}\left(1-\eta_t\right) = \frac{1}{\kappa^p\left(\mu^p-1\right)} \pi_t\left(1+\pi_t\right) - \frac{1}{1+r_t} \frac{1}{\kappa^p\left(\mu^p-1\right)} \pi_{t+1}\left(1+\pi_{t+1}\right) \frac{Y_{t+1}}{Y_t}$$

Further, (A.147) implies

$$1-\eta_t = 1 - \frac{mc_t - 1}{\frac{1}{\mu^p} - 1} = \frac{\frac{1}{\mu^p} - mc_t}{\frac{1}{\mu^p} - 1} = \frac{1 - \mu^p \cdot mc_t}{1-\mu^p} = \frac{\mu^p \cdot mc_t - 1}{\mu^p - 1}$$

Hence, we obtain the price Phillips curve

$$\pi_t\left(1+\pi_t\right) = \kappa^p\left(\mu^p \cdot mc_t - 1\right) + \frac{1}{1+r_t} \pi_{t+1}\left(1+\pi_{t+1}\right) \frac{Y_{t+1}}{Y_t}$$

which is equation (39) in the main text.

The first-order condition for $k_t$ is:

$$1 + \varphi'\left(\frac{k_t}{k_{t-1}}\right) = \frac{1}{1+r_t} \frac{\partial J_{t+1}}{\partial k_t} \equiv Q_t$$

and the envelope condition is:

$$\frac{\partial J_t}{\partial k_{t-1}} = mc_t \cdot F_k\left(k_{t-1}, n_t\right) - (1-\delta) - \varphi\left(\frac{k_t}{k_{t-1}}\right) + \varphi'\left(\frac{k_t}{k_{t-1}}\right) \frac{k_t}{k_{t-1}}$$

Rewriting and using (A.145), the first is

$$\frac{1}{\delta \epsilon_I}\left(\frac{k_t}{k_{t-1}} - 1\right) = Q_t - 1 \tag{A.150}$$

and the second is, using $\varphi'\left(\frac{k_t}{k_{t-1}}\right) \frac{k_t}{k_{t-1}} = \left(Q_t - 1\right) \frac{k_t}{k_{t-1}}$,

$$\left(1+r_{t-1}\right) Q_{t-1} = mc_t \cdot F_k\left(k_{t-1}, n_t\right) - \left(\frac{k_t}{k_{t-1}} - (1-\delta)\right) - \varphi\left(\frac{k_t}{k_{t-1}}\right) + \frac{k_t}{k_{t-1}} Q_t \tag{A.151}$$

where $F_k\left(k_{t-1}, n_t\right) = \alpha \frac{Y_t}{k_{t-1}}$ with Cobb-Douglas production.

To conclude, given the sequences $\left\{Y_t, \frac{W_t}{P_t}, r_t\right\}$, all firms starting the same initial level of capital $k_{-1} = K_{-1}$ choose the same sequence of labor $n_t = N_t$, capital $k_t = K_t$, and prices $\mathcal{P}_t = P_t$. Given the Cobb-Douglas production function, the time path for aggregates $\{N_t, K_t, Q_t, mc_t, \pi_t, I_t, J_t, p_t, d_t\}$ must satisfy:

a) The labor first-order condition (A.146), which reads:

$$mc_t = \frac{W_t}{P_t}\frac{1}{F_n\left(K_{t-1}, N_t\right)} = \frac{1}{1-\alpha}\frac{W_t N_t}{P_t Y_t} \tag{A.152}$$

b) The investment first-order condition (A.150), and the dynamics of $Q$, (A.151), which reads, for all for all $t \geq 1$:

$$\left(1 + r_{t-1}\right) Q_{t-1} \quad = \quad mc_t \cdot \alpha \frac{Y_t}{K_{t-1}} - \frac{I_t}{K_{t-1}} - \varphi\left(\frac{K_t}{K_{t-1}}\right) + \frac{K_t}{K_{t-1}} Q_t \tag{A.153}$$

c) The definition of investment,

$$I_t = K_t - (1-\delta) K_{t-1} \tag{A.154}$$

d) The production constraint, determining labor given target production and capital:

$$N_t = \left(\frac{Y_t}{\Theta} \cdot \frac{1}{K_{t-1}^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{A.155}$$

e) The new Keynesian Phillips curve (39)

f) The stock market valuation condition, with the ex-dividend price $p_t \equiv J_t - d_t$ satisfying, for all $t \geq 0$,

$$p_t = \frac{1}{1+r_t}\left(d_{t+1} + p_{t+1}\right) \tag{A.156}$$

and the date-$t$ dividend being given by:

$$d_t = F\left(K_{t-1}, N_t\right) - \frac{W_t}{P_t} N_t - I_t - \varphi\left(\frac{K_t}{K_{t-1}}\right) K_{t-1} - \xi\left(P_t, P_{t-1}\right) Y_t \tag{A.157}$$

## G.2 Additional model equations

The remaining equations for our quantitative model are:

a) Flow government budget constraint

$$G_t + \left(1 + r_{t-1}\right) B_{t-1} = T_t + B_t \tag{A.158}$$

b) Nonlinear wage Phillips curve (36)

c) Wage inflation consistency condition

$$1+\pi_t^w = (1+\pi_t)\frac{W_t/P_t}{W_{t-1}/P_{t-1}} \tag{A.159}$$

d) Market clearing equations:

$$\int\left(v_{it}^{liq}+v_{it}^{illiq}\right)di = 1 \tag{A.160}$$

$$\int\left(b_{it}^{liq}+b_{it}^{illiq}\right)di = B_t \tag{A.161}$$

$$\int c_{it}di+\zeta_t\int a_{it-1}^{liq}di+G_t+I_t+\varphi_t K_{t-1}+\xi_t Y_t = Y_t \tag{A.162}$$

where $\zeta_t \equiv \zeta(1+r_{t-1})$.

e) Fisher equation:

$$1+r_t=\frac{1+i_t}{1+\pi_{t+1}}$$

f) No arbitrage between bonds and shares, (32) at all $t\geq 0$.

**Walras's law.** Aggregating budget constraints, we have in the single-account models (A.113)

$$\int c_{it}di+p_t\int v_{it}di+\int b_{it}di = Z_t+(p_t+d_t)\int v_{it-1}di+(1+r_{t-1})\int b_{it-1}di$$

And in the two-account model (A.115), we have:

$$\begin{aligned}
& \int \widetilde{c}_{it}di+p_t\int\left(v_{it}^{liq}+v_{it}^{illiq}\right)di+\int\left(b_{it}^{liq}+b_{it}^{illiq}\right)di \\
= \; & Z_t+(p_t+d_t)\int\left(v_{it-1}^{liq}+v_{it-1}^{illiq}\right)di+(1+r_{t-1})\int\left(b_{it-1}^{liq}+b_{it-1}^{illiq}\right)di \\
& -\zeta(p_t+d_t)\int v_{it-1}^{liq}di-\zeta(1+r_{t-1})\int b_{it-1}^{liq}di
\end{aligned}$$

Defining $C_t \equiv \int c_{it}di$ in the single-account models, and $C_t \equiv \int \widetilde{c}_{it}di+\zeta(p_t+d_t)\int v_{it-1}^{liq}di+\zeta(1+r_{t-1})\int b_{it-1}^{liq}di$ in the two-account model, and using share market clearing (A.160) and bond market clearing (A.161) at all $t$, we obtain at every $t\geq 0$,

$$C_t+p_t+B_t=Z_t+p_t+d_t+(1+r_{t-1})B_t$$

Canceling $p_t$ on both sides, and using the government budget constraint (A.158), we obtain:

$$C_t+G_t=Z_t+T_t+d_t$$

Using the expression for dividends in (A.157), we obtain:

$$C_t + G_t + I_t + \varphi_t K_{t-1} + \xi_t Y_t = Z_t + T_t + Y_t - \frac{W_t}{P_t} N_t$$

Finally, using the definition of aggregate post-tax income, $Z_t = \frac{W_t}{P_t} N_t - T_t$, we obtain the goods market clearing condition:

$$C_t + G_t + I_t + \varphi_t K_{t-1} + \xi_t Y_t = Y_t$$

**Reducing to a single asset market clearing condition.** Since the no arbitrage condition (32) holds in equilibrium, we can apply the result of section F.1 to show that given paths $\{Z_t, r_t\}$, an initial set $\left(\omega_{i-1}^{liq}, \omega_{i-1}^{illiq}\right)$ of fractions invested in shares in each account, and initial values for $p_0 + d_0$, the policies $a_{it}^{liq}$, $a_{it}^{illiq}$, $c_{it}$ and the distribution over the total asset positions $D_t\left(a^{liq}, a^{illiq}\right)$ are both well defined, with agents indifferent between investing in bonds and shares.

Suppose that aggregate asset market clearing holds, ie:

$$A_t\left(\{Z_t, r_s\}, p_0 + d_0\right) = B_t + p_t \tag{A.163}$$

This implies that there is a path for individual portfolio shares $\omega_{it}$ in one-account models (and $\omega_{it}^{liq}, \omega_{it}^{illiq}$ in the two-account model), such that market clearing holds for each asset separately, ie

$$\begin{aligned} \int \left(1 - \omega_{it}^{liq}\right) a_{it}^{liq} di + \int \left(1 - \omega_{it}^{illiq}\right) a_{it}^{illiq} di &= B_t \\ \int \omega_{it}^{liq} a_{it}^{liq} di + \int \omega_{it}^{illiq} a_{it}^{illiq} di &= p_t \end{aligned}$$

while many such paths are possible, given that households are indifferent between holding bonds or shares in each account, one solution that works is $\omega_t = \frac{p_t}{A_t}$ for every $i$ and every $t$. In our calibration, we assume that $\omega_{-1} = \frac{p}{A}$ for every $i$ in the steady state. In summary, it is sufficient to enforce (A.163) and an initial $\omega_{i-1}$ consistent with market clearing to obtain equilibrium in both asset markets.

## G.3 Steady state

In a steady state, aggregates $\left\{Y_t, K_t, N_t, I_t, C_t, A_t, B_t, Q_t, p_t, d_t, mc_t, \frac{W_t}{P_t}, r_t, i_t, \pi_t, \pi_t^w\right\}$ are constant. We restrict our attention to the steady state with no inflation, $\pi = 0$. The NKPC (39) implies

$$mc = \frac{1}{\mu^p}$$

and price adjustment costs are $\xi = 0$. The first-order condition for investment (A.150) implies $Q = 1$, and the steady-state investment condition (A.154) implies $I = \delta K$. Given that capital adjustment costs are $\zeta = 0$, the first-order conditions for labor and capital (A.152), (A.153), and the production function (A.155), imply

$$\frac{WN}{PY} = \frac{1-\alpha}{\mu^p} \qquad \frac{K}{Y} = \frac{1}{\mu^p}\frac{\alpha}{r+\delta} \qquad \frac{1}{\Theta} = \left(\frac{K}{Y}\right)^{\alpha}\left(\frac{N}{Y}\right)^{1-\alpha} \tag{A.164}$$

Given our calibration for $r, \alpha, \delta, \mu^p$ in tables 2 and 3 and our normalization $Y = N = 1$, equations (A.164) imply $\frac{W}{P}$, $\frac{K}{Y}$ and $\Theta$. The steady-state versions of (A.156) and (A.157) also imply

$$\begin{aligned} p = \frac{d}{r} &= \frac{Y}{r}\left(1 - \frac{W}{P}\frac{N}{Y} - (r+\delta)\frac{K}{Y} + r\frac{K}{Y}\right) \\ p &= K + \frac{Y}{r}\left(1 - \frac{1}{\mu^p}\right) \end{aligned} \tag{A.165}$$

showing that the steady-state stock price incorporates both the value of the capital stock and the capitalized value of markups.

The government budget constraint at steady state reads

$$G = rB + T \tag{A.166}$$

Given our calibration for $G/Y$ and $B/Y$ in table 3 and our normalization $Y = 1$, we obtain $G, B$, and $T$ from (A.166).

On the household side, given our calibration of one of the models described in Table 2, post-tax income $Z = \frac{W}{P}N - T$, and $p + d = (1+r)\,p$, we obtain the steady-state consumption, asset, and virtual-consumption functions:

$$C = \mathcal{C}\left(\{Z, r, \beta\}, (1+r)\,p\right) \qquad A = \mathcal{A}\left(\{Z, r, \beta\}, (1+r)\,p\right) \qquad C^* = \mathcal{C}^*\left(\{Z, r, \beta\}, (1+r)\,p\right) \tag{A.167}$$

The wage consistency condition (A.159) implies that there is no wage inflation, $\pi^w = 0$, which from (36) implies

$$\frac{\gamma N^{\frac{1}{\phi}}}{\left(C^*\right)^{-\sigma}(1-\theta)\frac{Z}{N}} = \frac{1}{\mu^w} \tag{A.168}$$

We calibrate $\mu^w = 1$ and the progressivity parameter $\theta$, and $\gamma$ follows from equation (A.168).

Finally, the asset market clearing condition (A.163) is

$$\mathcal{A}\left(\{Z, r, \beta\}, (1+r)\,p\right) = B + p = B + K + \frac{Y}{r}\left(1 - \frac{1}{\mu^p}\right) \tag{A.169}$$

and the goods market clearing condition (A.162) is

$$\mathcal{C}\left(\{Z, r, \beta\}, (1+r)\,p\right) + G + I = Y \tag{A.170}$$

Given all of our other calibration targets, for any model of consumption, we find $\beta$ that satisfies (A.169), and make sure that (A.170) is also satisfied, as it should given Walras's law.

In a steady state there are no valuation effects, so $\mathcal{A}(\{Z, r, \beta\}, (1+r)p) = \mathcal{A}(\{Z, r, \beta\})$. The models of section 2–5 are calibrated to have identical $A/Z$ as in the quantitative model.

### G.4 Additional model simulations

This section presents the additional model simulations mentioned in the main text. Figure G.1 compares responses in the HA-two model across degrees of deficit financing $\rho_B$, finding that greater $\rho_B$ leads to unambiguously stronger output and consumption responses.

Figure G.2 reproduces figure G.1 in the TABU model. As claimed, consumption multipliers are never positive in the TABU model, and the impulse response is not monotone in $\rho_B$. This is due to the effect of the decline in the stock market on consumption. This effect is implausibly large because of the very high iMPCs out of capital gains $\mathbf{m}^{cap}$ of the TABU model.

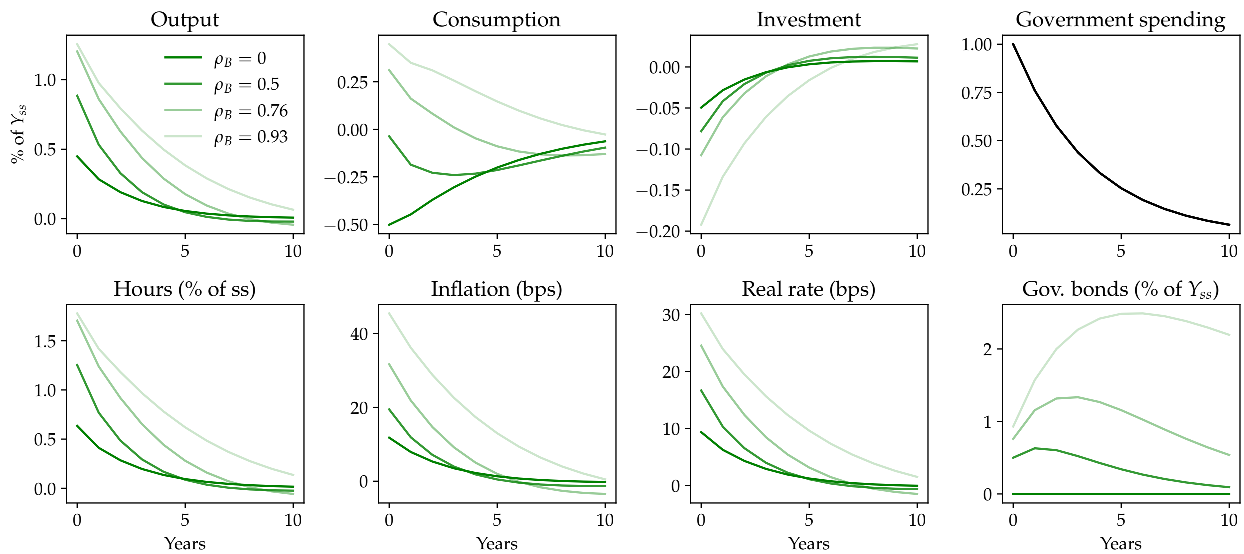
Figures G.3–G.6 reproduce figure G.1 with $\rho_B = 0.93$, varying one parameter at a time in the HA-two model. Figure G.3 varies the Taylor rule coefficient $\phi_\pi$. Provided $\phi_\pi$ is above the HA-two determinacy threshold of 1.05, higher $\phi_\pi$ raises real rates and amplifies crowd-out of investment and consumption, but consumption still rises in fiscal expansion unless $\phi_\pi$ is around 2. Note that at $\phi_\pi = 1.1$ investment is essentially constant, as the dampening effect from the rising user cost of capital is offset by the expansionary effect from rising marginal product of labor.

Figure G.4 shows the impulse response as we vary the elasticity of investment to $Q$, $\epsilon_I$, around its benchmark value of 4. When investment is more sensitive to $Q$, the crowding-out effect is larger and equilibrium real interest rates rise by less. While consumption multipliers remain positive for most plausible values of $\epsilon_I$, as $\epsilon_I$ gets very large ($\epsilon_I \simeq 20$), the negative effect on labor incomes from declining investment starts to be too large and consumption actually falls.

Figure G.5 shows the impulse responses as we vary the slope of the price Phillips curve $\kappa^p$. We do this by taking the formula from the main text, $\kappa^p = \frac{1}{1+\Gamma^p}(1-\beta(1-\text{freq}))\,\text{freq}/(1-\text{freq})$ with a annual frequency of price reset freq $= 0.67$ from Nakamura and Steinsson (2008), and varying the degree of real rigidity $\Gamma^p \in \{0, 1, 5, 9\}$. Here, $\Gamma^p = 0$ corresponds to the standard Calvo formula without real rigidity and $\Gamma^p = 5$ is our benchmark. As prices become more flexible for given wage rigidity, the consumption multiplier rises. Intuitively, with more flexible prices, real wages get closer to the marginal product of labor, which falls with output, and this reduces post-tax labor incomes and therefore consumption. Counterbalancing this, margins—and so the stock market—fall by less, but this has only a small effect on consumption because of the low MPC out of capital gains. All in all, with more flexible prices, the intertemporal Keynesian cross effect described in sections 2–5 is dampened by an offsetting movement in the real wage.

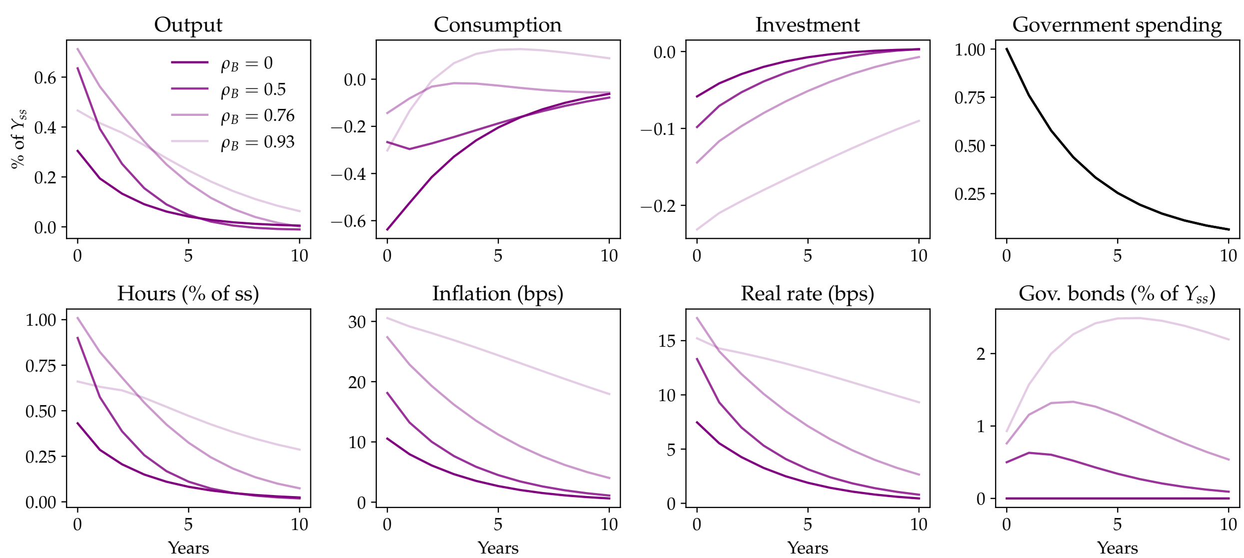
Figure G.6 shows the impulse responses as we vary the slope of the wage Phillips curve $\kappa^w$. We do this by taking the formula from the main text, $\kappa^w = \frac{1}{1+\Gamma^w}(1-\beta(1-\text{freq}))\,\text{freq}/(1-\text{freq})$ with a annual frequency of wage reset freq $= 0.33$ from Grigsby et al. (2021), and varying the degree of real rigidity $\Gamma^w \in \{0, 1, 5, 9\}$. Again, $\Gamma^w = 0$ corresponds to the standard Calvo formula without real rigidity and $\Gamma^p = 5$ is our benchmark. Here again, rising wage flexibility dampens the consumption multiplier, with similar magnitudes as with rising price flexibility.

Figure G.1: Response to a government spending shock in our quantitative two-account model HA-two



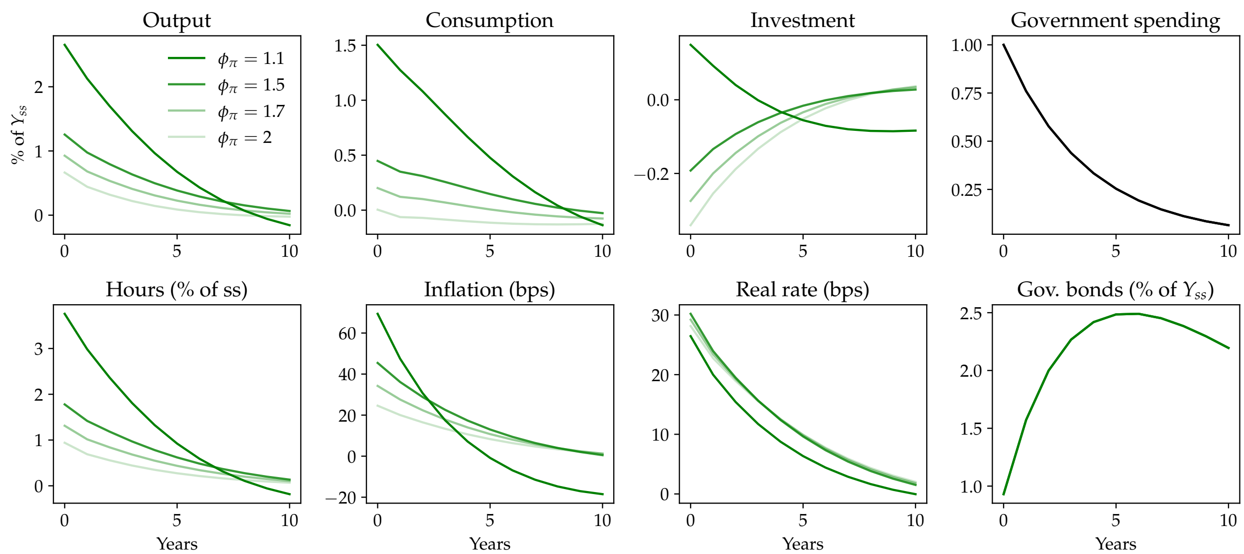

*Note:* The HA-two model is a two-account heterogeneous-agent model which is calibrated to match evidence on intertemporal MPCs. The government spending shock declines exponentially at rate $\rho_G = 0.76$ and public debt follows $dB_t = \rho_B (dB_{t-1} + dG_t)$ for various values of $\rho_B$.

Figure G.2: Impulse responses as a function of $\rho_B$ in the TABU model



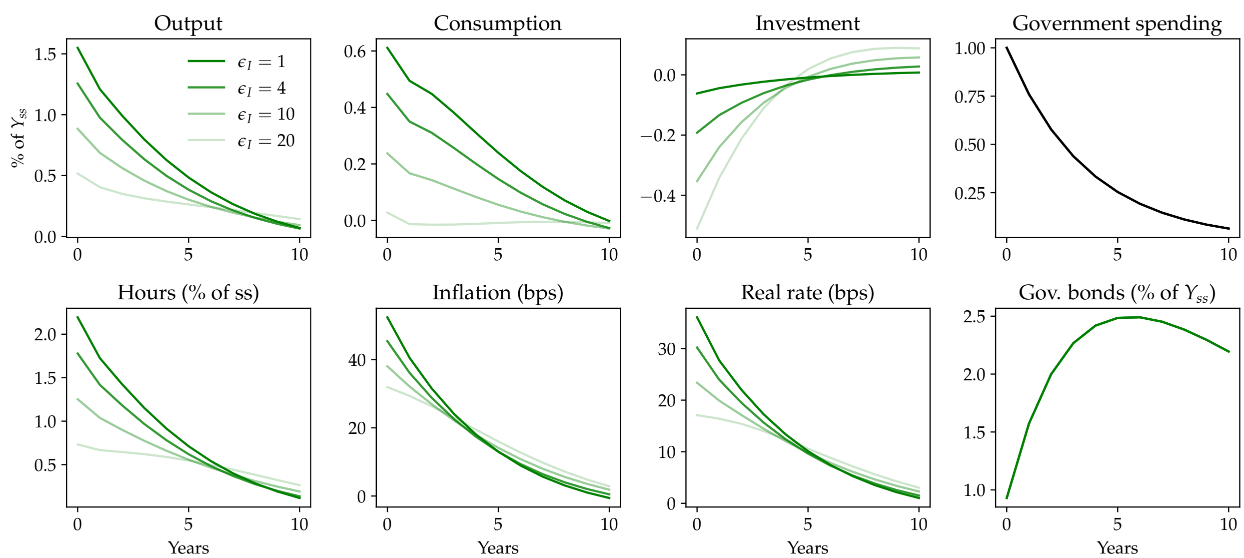

*Note:* This figure reproduces figure G.1 but in the TABU model.

Figure G.3: Impulse responses as a function of the Taylor rule coefficient $\phi_\pi$



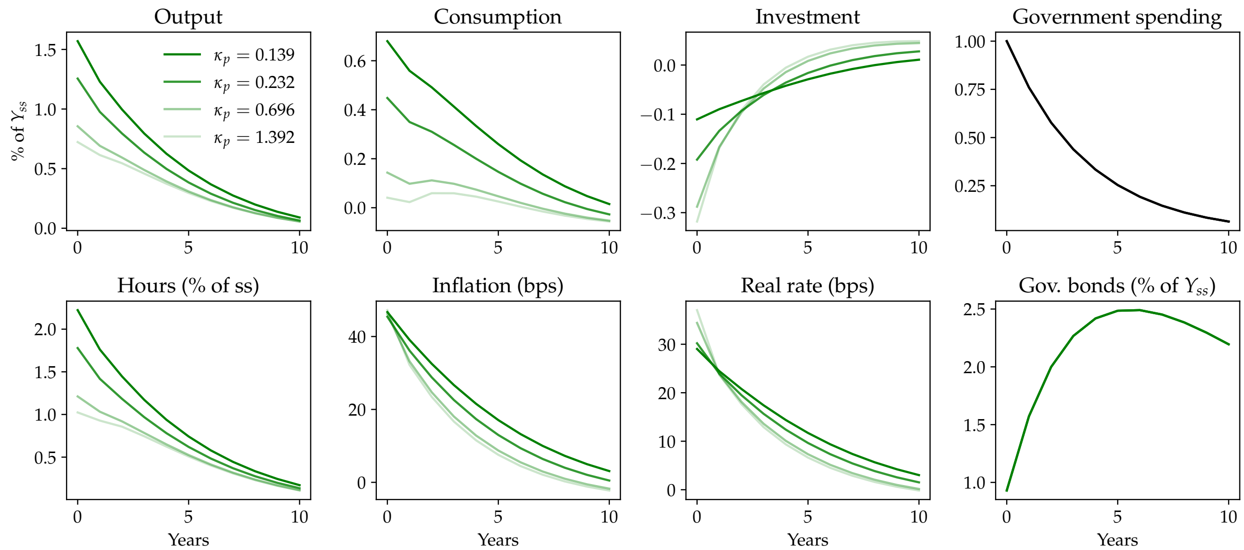

*Note:* This figure reproduces figure G.1 with $\rho_B = 0.93$, and varies the level of the Taylor rule coefficient $\phi_\pi$. The benchmark calibration has $\phi_\pi = 1.5$ (see table 3).

Figure G.4: Impulse responses as a function of the investment sensitivity parameter $\epsilon_I$



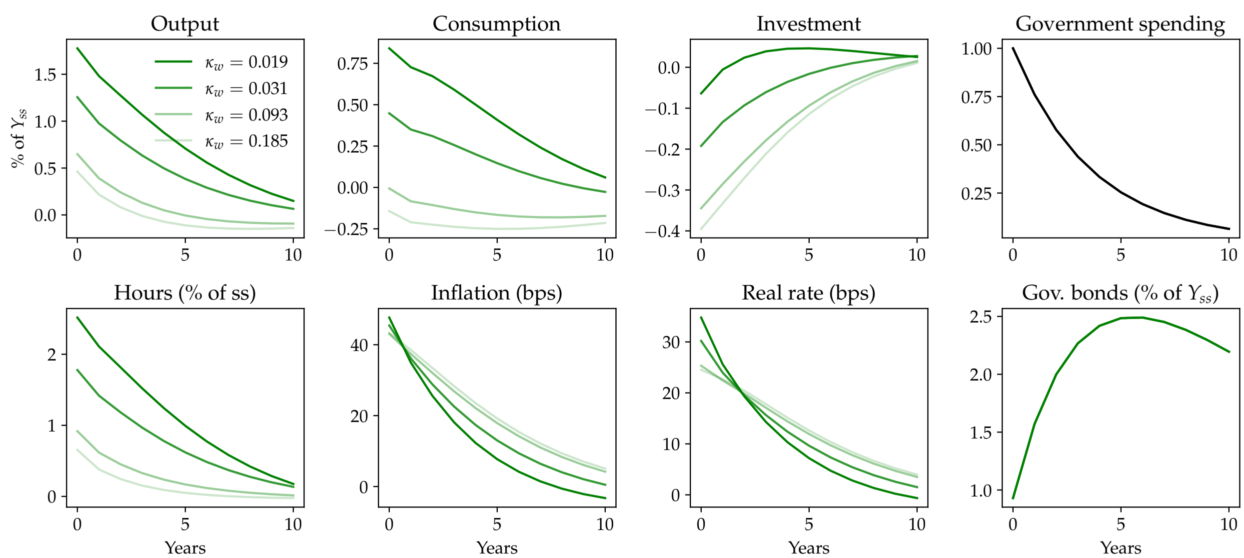

*Note:* This figure reproduces figure G.1 with $\rho_B = 0.93$, and varies the level of the investment sensitivity parameter $\epsilon_I$. The benchmark calibration has $\epsilon_I = 4$ (see table 3).

Figure G.5: Impulse responses as a function of the slope of the price Phillips curve $\kappa^p$




*Note:* This figure reproduces figure G.1 with $\rho_B = 0.93$, and varies the slope of the price Phillips curve $\kappa^p = \frac{1}{1+\Gamma^p}(1-\beta(1-\text{freq}))\,\text{freq}/(1-\text{freq})$ by altering the degree of real rigidity $\Gamma^p \in \{0, 1, 5, 9\}$, with freq $= 0.33$ as in the main text. The benchmark calibration has $\Gamma^p = 5$ (see section 7.1).

Figure G.6: Impulse responses as a function of the slope of the wage Phillips curve $\kappa^w$




*Note:* This figure reproduces figure G.1 with $\rho_B = 0.93$, and varies the slope of the price Phillips curve $\kappa^w = \frac{1}{1+\Gamma^w}(1-\beta(1-\text{freq}))\,\text{freq}/(1-\text{freq})$ by altering the degree of real rigidity $\Gamma^w \in \{0, 1, 5, 9\}$, with freq $= 0.33$ as in the main text. The benchmark calibration has $\Gamma^w = 5$ (see section 7.1).