Taxing Sudden Capital Income Jumps

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New Fortunes

- By examining 100 of the richest Americans listed in the Forbes magazine, Graham (2021) finds
  - "[b]y 2020 the biggest source of new wealth was what are sometimes called 'tech' companies. Of the 73 new fortunes, about 30 derive from such companies. These are particularly common among the richest of the rich: 8 of the top 10 fortunes in 2020 were new fortunes of this type."
- Halvorsen, Hubmer, Ozkan, and Salgado (2023) examine the Norwegian administrative data
  - at least a quarter of wealthiest (top 0.1%) people start with debt but experience rapid wealth growth early in life as there were some sudden large new fortunes generated from private equity investments.

What We Do

- Depart from the standard incomplete market model, derive analytical solution to highlight how capital income jump risks affect the wealth distribution
  - recursive utility generates realistic MPC
  - tractable aggregation / recursive moments of earnings and wealth distributions
  - introduce hyper-exponential distribution (HED) for the jump size that generates a realistic wealth distribution
- Explore the implication of capital taxation, including taxing the jump income (e.g., progressive capital taxation)
  - taxing the jump income can be more efficient
  - transfer tax revenues can increase inequality
  - taxes to finance additional gov. debt can reduce inequality

Literature

- Investment/entrepreneur risks

  - Quadrini (2000), Cagetti and De Nardi (2006), Angeletos and Calvet (2006), Angeletos (2007), Benhabib, Cui, and Miao (2024)...

- Different taxation results compared to random-return (Keston process and thus Pareto tail) models such as Benhabib et al. (2011), Guvenen et al. (2023). The rare nature of income surges means that investment incentive is inelastic to progressive tax.

- Progressive capital income tax, different from optimal progressive tax schedule in labor income: Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009), Bakis (2015, Heathcote et al. (2017). Related to taxing the extreme high earnings in Kindermann and Kruger (2022).

- Debt policy redistribution related to Aiyagari and McGrattan (1998), Angeletos, Collard, and Dellas (2020), Bayer, Born, and Luetticke (2023), and Bassetto and Cui (2023). But the tax revenues aspect is new.

Environment

- A continuum of infinitely-lived households endowed with average L units of labor, supplied inelastically in a competitive labor market.
- Each household owns and runs a private firm, which employs labor but can only use the capital stock invested by the particular household.
- Two sources of idiosyncratic shocks hit its private firm and labor earnings independently.
- Only trade riskless private and government bonds and cannot fully diversify away idiosyncratic shocks.
- We focus on stationary economy.
- Continuous-time affine jump-diffusion (AJD) framework.

Preferences and Technology

- Epstein-Zin-Duffie recursive utility

$$U_t = f^{-1}\left(f(c_t)dt + \exp(-\beta dt) f(\mathcal{R}_t(U_{t+dt}))\right)$$

Weil (1993) mixed CRRA + CARA specification:

$$f(c) = \frac{c^{1-1/\psi}}{1-1/\psi}, \quad \mathcal{R}_t(U_{t+dt}) = u^{-1}\mathbb{E}_t u(U_{t+dt}), \quad u(x) = \frac{-\exp(-\gamma x)}{\gamma}$$

- ψ EIS parameter, γ absolute risk aversion

- Production function: $y_t = Ak_t^{\alpha} n_t^{1-\alpha}$. (After-tax) Profit maximization implies

$$R^k k_t = (1-\tau_k) \max_{n_t} \left\{ Ak_t^{\alpha} n_t^{1-\alpha} - \frac{w}{1-\tau_\ell} n_t - \delta k_t \right\}$$

Labor and Capital Income Risks

- Labor supply process (similar to CIR interest-rate process):

$$d\ell_t = \rho_\ell (L - \ell_t)\, dt + \sigma_\ell \sqrt{\ell_t} dW_t^\ell,$$

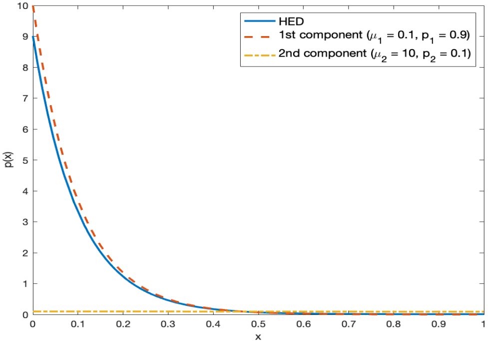
- Entrepreneurial profits (capital income) π_t follow dynamics

$$d\pi_t = R^k k_t dt - \left(\chi k_t + \frac{\eta}{2} k_t^2\right) dt + dJ_t.$$

- J_t: jump process with random jump size q following hyper-exponential distribution (HED) and with intensity $\lambda_t = \lambda_k k_t$.
- $\left(\chi k_t + \frac{\eta}{2} k_t^2\right)$ captures maintenance costs (illiquidity)

Example HED Probability Density Function

$$f(q) = \sum_{j=1}^{n} p_j \frac{\exp\left(-x/\mu_j\right)}{\mu_j}, \; x > 0, \; \mu_j = (1-\tau_k)(1-\tau_J)\tilde{\mu}_j$$



Budget Constraints

- Denote $x_t = k_t + b_t$, and b includes both private and public debt. Budget constraint:

$$dx_t = rb_t dt + d\pi_t + w\ell_t dt - c_t dt + \Upsilon dt,$$

where Υ represents per capita government transfers (or lump-sum taxes if $\Upsilon < 0$).

- Combining yields

$$\begin{aligned} dx_t = & \, rx_t dt + \left(R^k - \chi - r\right) k_t dt - 0.5\eta k_t^2 dt + dJ_t \\ & + w\ell_t dt - c_t dt + \Upsilon dt \end{aligned} \tag{1}$$

- No binding borrowing constraints

Household's Problem

- Let $V(x,\ell)$ denote the value function. Guess and verify the value function, $V(x,\ell) = \theta(x + \xi_\ell \ell + \xi_0)$. Then, HJB equation:

$$\beta f(V_t) = \max_{c_t, k_t} f(c_t) + f'(V_t)\Big[\mu_t + 0.5\frac{u''(V_t)}{u'(V_t)}(\sigma_t^W)^2 + \lambda_t \frac{\mathbb{E}_\nu[u(V_t + \sigma_t^J) - u(V_t)]}{u'(V_t)}\Big]$$

  with μ_t, σ_t^W, and σ_t^θ from the Ito's Lemma.
- Why CARA? For aggregation;
- Why CRRA? For non-negative consumption and wealth distribution (MPC);
- Why CIR earning process? For tractability and non-negative labor income - simple simulation with importance sampling when looking at income growth distribution

Household's Problem - Solution

Following BCM (2024) result and apply HED, the optimal consumption rule is given by

$$c_t = \theta^{1-\psi}\left(x_t + \xi_\ell \ell_t + \xi_0\right) = \vartheta\left(x_t + a_h h_t + \Gamma\right), \tag{2}$$

and the capital demand is given by

$$k_t = k \equiv \frac{1}{\eta}\left(R^k - \chi - r + \lambda_k \sum_j \frac{p_j}{\mu_j^{-1} + \gamma\theta}\right), \tag{3}$$

where $\vartheta = \psi(\beta - r) + r$ and $\theta = \vartheta^{1/(1-\psi)}$, $\Gamma = \eta k^2/(2r)$, and h_t is "human wealth"

Solution Interpretation

- $\vartheta = \psi(\beta - r) + r$ is MPC, above r for $\beta > r$; EIS ψ is important
- h_t is present value of future labor income
- $\Gamma = \eta k^2 / (2r)$ can be rewritten as

$$\Gamma = \frac{1}{r}\left\{\left(R^k - \chi - r\right) k + \frac{\lambda_k k}{\gamma\theta}\mathbb{E}_\nu\left[1 - \exp\left(-\gamma\theta q\right)\right] - \frac{\eta}{2}k^2 + \Upsilon\right\},$$

  according to (3).
- Γ is essentially the present value of expected (risk- and cost- adjusted) profits from the capital investment.
- The risk adjustment captures precautionary savings against capital jump risks.

Stationary Equilibrium (skip)

- Given fiscal policy $(\tau_k, \tau_\ell, \tau_J, G, B)$, a stationary competitive equilibrium consists of constant wage w and capital return R^k, individual choices $\left\{c_t^i, k_t^i\right\}_{t\geq 0}$ for $i \in [0,1]$, and constant aggregate quantities consumption, output, and capital such that
  - (i) given $\left(w, R^k\right)$, the processes $\left\{c_t^i, k_t^i\right\}_{t\geq 0}$ are optimal choices for each household i;
  - (ii) the bond and labor markets clear;
  - (iii) the government budget constraint holds

$$G + Y + rB = \frac{\tau_k}{1-\tau_k}\left(R^k + \frac{\tau_J}{1-\tau_J}\lambda_k \mathbb{E}_\nu[q]\right)K + \frac{\tau_\ell}{1-\tau_\ell}wL, \tag{4}$$

Equilibrium Wealth Distribution

- Solve for the aggregate equilibrium first, and then characterize and simulate the cross-sectional distribution.
- Substitute the optimal consumption rule (2) into the wealth dynamics:

$$dx_t = -\rho_x x_t dt + \mu_x dt + \phi w \ell_t dt + dJ_t, \tag{5}$$

where

$$\rho_x \equiv \psi(\beta - r), \quad \phi \equiv 1 - \frac{\vartheta \xi_\ell}{w}. \tag{6}$$

Clearly, $\rho_x > 0$ if $r < \beta$. The term ϕ represents the marginal propensity to save (MPS) out of labor income. We restrict to equilibrium with $\phi > 0$.

- Joint with ℓ_t, a system of AJD processes

A Few Properties

- Denote $\zeta_j \equiv \mathbb{E}_\nu \left[q^j\right]$. Then

$$\frac{\mathrm{Var}[x]}{\mathrm{Var}[z]} = \frac{\phi^2}{\rho_x\left(\rho_x+\rho_\ell\right)} + \frac{\lambda_k K \zeta_2}{2\rho_x \mathrm{Var}[z]},$$

$$\frac{\mathrm{Skew}[x]}{\mathrm{Skew}[z]} = \frac{2\sqrt{\rho_x\left(\rho_x+\rho_\ell\right)}}{2\rho_x+\rho_\ell}\left[1+\frac{\left(\lambda_k K\zeta_2\right)\left(\rho_x+\rho_\ell\right)}{2\mathrm{Var}[z]\phi^2}\right]^{-3/2} + \frac{\lambda_k K\zeta_3}{3\rho_x \mathrm{Skew}[z]\left(\mathrm{Var}[x]\right)^{3/2}},$$

$$\frac{\mathrm{Kurt}[x]}{\mathrm{Kurt}[z]} = \dots$$

- Both the stationary wealth and labor income distributions have an exponential tail with the exponential decay rates given by

$$\overline{\alpha}_x \equiv \min_j\left\{\frac{1}{(1-\tau_k)(1-\tau_J)\tilde{\mu}_j}\right\}, \quad \overline{\alpha}_z \equiv \frac{2\rho_\ell}{\sigma_\ell^2}.$$

A Few Properties: some comments

- Jump risk does not necessarily generate a more positively skewed and heavier tailed wealth distribution than labor income distribution
- You need just enough illiquidity; savings cannot be too strong, or too weak
- In skewness comparison, the first term shows that jump risk reduces the wealth skewness relative to the earnings skewness. However, the second term increases the wealth skewness when $\zeta_3 > 0$.

- If capital and bonds are perfect substitutes (i.e., $\lambda_k = \chi = \eta = 0$), this proposition is reduced to findings of Wang (2007).
  - $Skew[x] < Skew[z]$
  - Labor income jumps *cannot* resolve the issue in Wang (2007).

Quantitative Analysis: calibration

Table: Calibrated Parameter Values

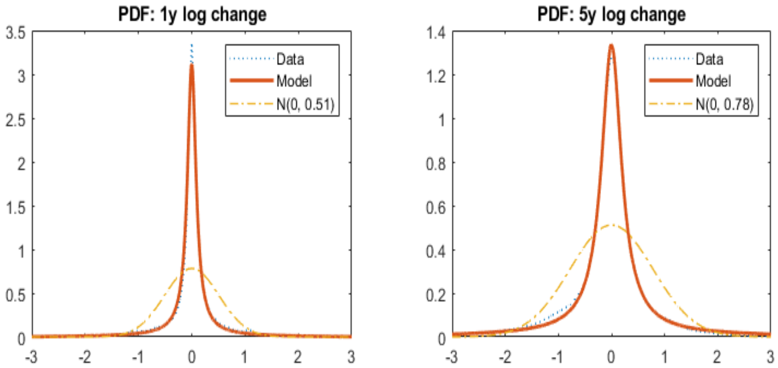
	Value	Explanation/Target		Value	Explanation/Target
β	0.1417	MPC $= 0.20$	B	1.6101	$B/Y = 0.81$
γ	4.1	relative risk aversion 5	G	0.3777	$G/Y = 0.19$
ψ	1.5	EIS	μ_2	414.54	top 0.1% wealth share
α	0.33	capital share	μ_1	0.1050	top 20% wealth share
δ	0.1251	$I/Y = 0.16$	p_2	0.0048	average innovation return 14%
A	1.3120	$w = 1$	p_1	0.9952	$1 - p_2$
L	0.8000	estimated	η	0.0049	$R^k - r = 3.0\%$
ρ_ℓ	0.0030	estimated	χ	0.0175	interest rate $r = 2.5\%$
σ_ℓ	0.1097	estimated	λ_k	0.05	innovation probability
$\tau_\ell = \tau_k$	0.25	average tax rate			

Quantitative Analysis - moments of earnings growth (SMM)

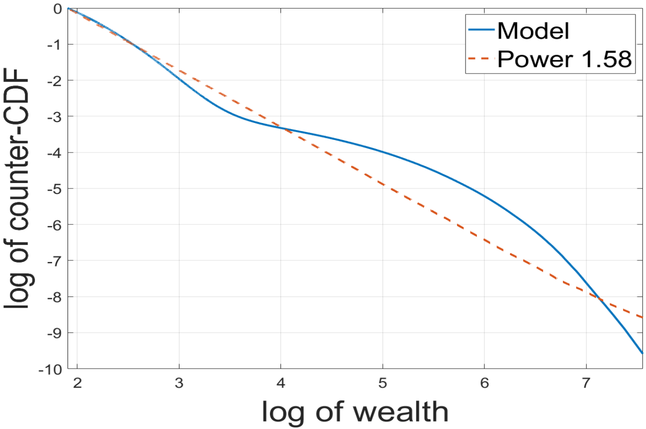
Δ log-earnings	Std. Dev.	Skewness	Kurtosis	fraction $< 5\%$	$< 10\%$	$< 20\%$
Data	0.51	- 1.07	14.93	30.6%	48.8%	66.5%
Model	0.51	- 0.03	15.98	29.1%	48.9%	68.8%

Note: Simulation results are from SMM estimates $\rho_\ell = 0.0033$, $\sigma_\ell = 0.1073$, and $L = 0.7775$. Given a set of the 3 parameters, the simulation has 10^5 agents who start with levels of earnings drawn from the invariant distribution implied by the earning process; then, we simulate the earning for each of the agents for 2 years, and we calculate the moments of the cross-sectional distribution of log (annual) earning changes; the calculation is repeated for 100 times and we take averages. A minimizer routine searches for the parameters that minimize the distance between the model and the data. Each dt is approximated by one week, which is 1 / 52, as a year has roughly 52 weeks. Therefore, given parameters, the simulation has $10^5 \times 100 \times 104 = 1.04 \times 10^9$, roughly 1 billion, person-week observation.

Earning Growth from Guvenen et. al. (2021) data v.s. Model



Quantitative Analysis: wealth distribution tail comparison



Quantitative Analysis: wealth distribution

Wealth	Top 0.1%	1%	10%	20%	50-10%	Bottom 50%	Gini
Net worth	15%	31.5%	66.7%	79.8%	32.0%	1.7%	0.80
Model	15%	32.2%	62.7%	79.9%	33.7%	1.5%	0.77

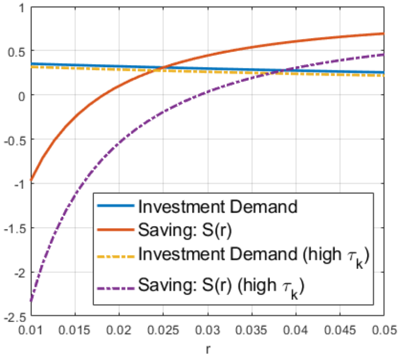
Note: The measure of top 0.1% share is from Smith et.al. (2020). Top 20% data is the average from survey of consumer finance after 2000. The rest are the averages between 2000 and 2019 obtained from distributional financial account of Federal Reserve Board. The model statistics is the average of 100 simulation of 15 years with dt approximated by one day, and each simulation has 100K people starting from the same initial level of labor income and wealth.

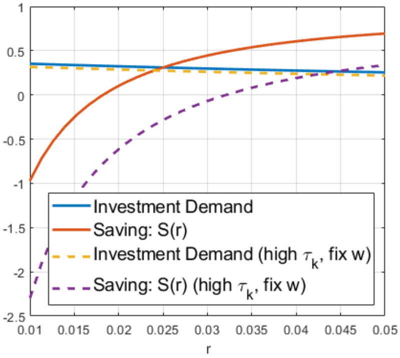
Policy Experiments: capital taxation

- Two types of capital taxation
  - Flat capital tax
  - Progressive tax on capital income jump
- Two types of spending policies given same $rB + \Upsilon$, holding G fixed
  - Raise lump-sum transfer Υ, holding B fixed
  - Raise debt B, holding Υ fixed
- Two types of spending policies have the same aggregate effects:
  - a kind of "Ricardian Equivalence"
  - same aggregate saving function, no effect on aggregate investment function

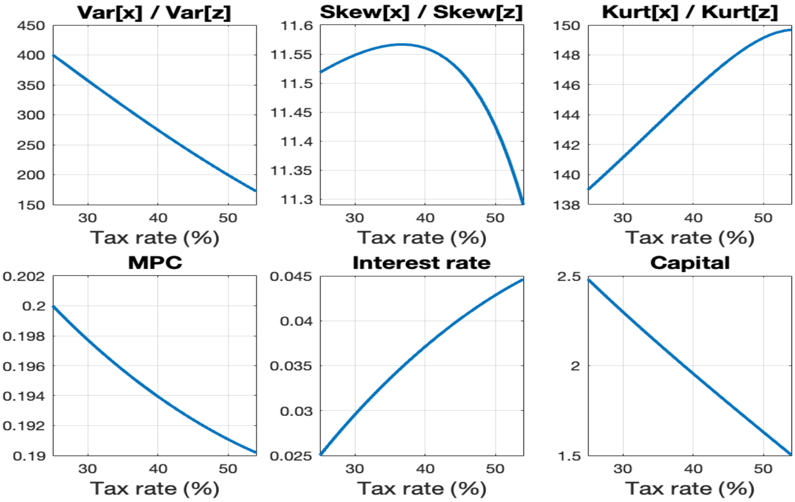
Investment and Saving Functions: after increasing tax

Our model has a (unique) equilibrium with $0 < r < \beta$. Under CM, $r = \beta$.





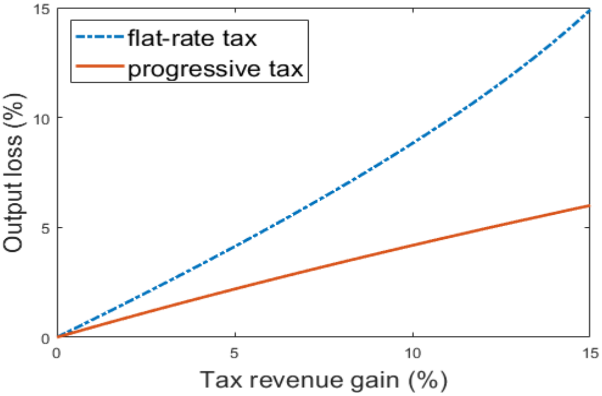
Aggregate Impact of Flat-rate Tax



Investment is Inelastic w.r.t. τ_J.

Recall the jump risk-premium term

$$\lambda_k \sum_i \frac{p_j}{((1-\tau_k)(1-\tau_J)\tilde{\mu}_i)^{-1} + \gamma\theta}$$



Capital Taxation: distributional effects 1

Raise 5% extra tax revenue

Table: Taxation with lump-sum transfer policy

	Capital	Wealth	$r(\%)$	MPC(%)	Bottom 50% (%)	Top 10% (%)	Top 1% (%)	Top 0.1% (%)	Gini Coeff. (%)
$\tau_k = 0.25,\ \tau_J = 0$	2.48	4.05	2.50	20.00	1.5	62.7	32.2	15.0	77.0
$\tau^k = 0.3115,\ \tau_J = 0$	2.26	3.94	3.05	19.72	0.50	63.4	32.0	15.3	77.5
$\tau^k = 0.25,\ \tau_J = 0.1373$	2.36	4.08	3.11	19.70	0.87	63.1	32.0	15.0	77.3

With no borrowing constraints, HH borrow more to raise consumption due to wealth effect, and thus inequality rises

Capital Taxation: distributional effects 2

Table: Taxation with bond policy

	Capital	Wealth	$r(\%)$	MPC(%)	Bottom 50% (%)	Top 10% (%)	Top 1% (%)	Top 0.1% (%)	Gini (%)
$\tau_k = 0.25,\ \tau_J = 0$	2.48	4.05	2.50	20.00	1.5	62.7	32.2	15.0	77.0
$\tau^k = 0.3115,\ \tau_J = 0$	2.26	4.33	3.05	19.72	4.88	58.7	29.4	14.0	73.2
$\tau^k = 0.25,\ \tau_J = 0.1373$	2.36	4.42	3.11	19.70	4.75	58.9	29.5	13.8	73.5

With more bond assets, inequality declines

Conclusion

- Develop a tractable macro framework with exponential-tailed wealth distribution
  - Capital is partially liquid and capital income jumps follow a HED
- Takeaways:
  - Illiquidity of investment and return jumps may generate a thicker wealth tail than earnings' tail
  - Taxing the jump income may be a good idea, both for the aggregate and for the distribution, if the redistribution is done carefully
- Future research
  - target transfers / optimal debt-management policy
  - monetary-fiscal interactions with $r < g$ and inequality
  - and more...